Analysis of Trade-offs between Buffer and QoS Requirements in Wireless Networks

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ABSTRACT

In this paper, we consider the scheduling problem where data packets from *K* input flows need to be delivered to *K* corresponding wireless receivers over a heterogeneous wireless channel. Our objective is to design a wireless scheduler that optimizes the buffer requirement at each wireless receiver while maintaining good throughput performance. This is a challenging problem due to the unique characteristics of the wireless channel.

We propose a novel idea of exploiting both the long-term and short-term error behavior of the wireless channel in the scheduler design. In addition to typical first-order Quality of Service (QoS) metrics such as throughput and delay, our performance analysis of the scheduler permits the evaluation of higher-order metrics, which are needed to evaluate the buffer requirement. We show that the proposed scheduler achieves high overall throughput as well as low buffer requirement when compared to other wireless schedulers that only make use of the instantaneous channel state in a heterogenous channel.

Keywords: Wireless Scheduling, QoS, Heterogenous Channel, Buffer Requirements

1 Introduction

We consider the problem where data packets from K input-flows need to be delivered to K corresponding wireless receivers via a wireless media. With the huge success of mobile telephony coupled with a phenomenal growth of internet users, one such scenario is depicted in the wireless network in the left hand side (LHS) of Fig. 1. We consider the downlink scheduling problem at access point B as shown in the right hand side (RHS) of Fig. 1.

The design of the wireless scheduler is an important problem in wireless networking for:

Wireless Application Development : In order to be meaningful, data packets must be delivered to each wireless receiver at specific data rates, and/or within specific delay, packet loss and jitter bounds. These requirements are collectively known as Quality of Service (QoS). Wireless scheduling is an important component of QoS provisioning over the wireless link, which determines if diverse applications such as multi-media messaging, voice-over-WLAN and localized-content distribution can be supported.

Wireless Receiver Design : The design of the wireless scheduler impacts the energy consumption and the buffer requirement, which are major considerations in the wireless receiver design. This is because of the limited memory capacity and battery power imposed by the size constraint of portable wireless devices.

While the capacity of a wired link is usually assumed to be constant, the following property makes the problem a harder and more challenging one:

Property 1 *A typical wireless link is characterized by:*

a. *High channel error rate*

b. *Bursty and time-varying channel capacity*

c. *Location-dependent channel capacity*

1.1 Related Work

The design of scheduling policies to meet QoS objectives over a wired link is a well-studied problem ([1, 2, 3], to name a few). Since these guarantees no longer hold over a wireless link, attempts were made to incorporate the effects of the channel characteristics into the guarantees. E.g., in [4], the authors studied the delay performance of a simple ARQ error control strategy for communications over a bursty channel for a *single* flow. In [5], the author investigated the characteristics and traffic effects of variable-rate communication servers. However,

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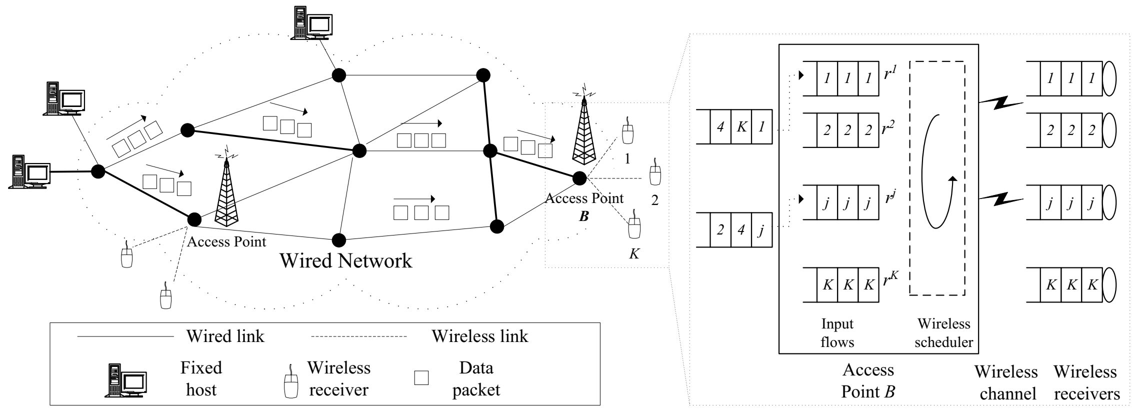

Figure 1: A generic wireless network where data packets are delivered to wireless receivers via access points (left) and an illustration of a wireless scheduling problem at an access point *B* (right).

the scheduling policy considered is not *channel-aware* since the channel is assumed to be location-independent. Channel-awareness is considered in the resource allocation problem in [6], where the authors characterized the stability properties of the system and proposed an optimal allocation policy that maximizes throughput and minimizes delay. However, the results apply only when the channel errors are time-uncorrelated.

An alternative approach is to utilize feedback from each receiver to predict the *instantaneous* channel state (i.e., whether it is erroneous or error-free) and the *long-term* behavior such as the burstiness of that channel. Due to characteristics (b) and (c) in Property 1, it is highly likely that at least one receiver with an error-free channel exists at any instant. Hence, channel efficiency can be optimized by restricting the candidates for transmission to those with *predicted* error-free channels in channel-state dependent (CSD) schedulers proposed in [7, 8]. In [9, 10], the authors considered the downlink scheduling problem in a CDMA system. In this case, the channel information is embedded in the measured data rates, and the authors proposed an exponential rule that optimizes the throughput.

A comprehensive survey of variants of CSD schedulers that differ in the mechanism of selecting the *instantaneous* 'best' flow to transmit while trading-off amongst various performance constraints such as throughput, fairness and delay can be found in [11]. In particular, the concept of 'compensation' was introduced in CSD schedulers proposed in [12, 13, 14, 15, 16, 17] to achieve a tradeoff between channel efficiency and *short-term fairness* provision. These schedulers can be mapped to the Unified Wireless-Fair Queueing (UWFQ) architecture proposed in [18]. In addition, the QoS performance of these schedulers in terms of first-order metrics such as throughput and delay are evaluated in this work.

1.2 Contributions of This Paper

In this paper, we propose a wireless scheduler that partitions the receivers according to the burstiness of its channel, and then applies a different scheduling mechanism to each partition. We present a detailed performance analysis of the proposed scheduler using the framework from our earlier work [19], and show that it achieves a good balance between wireless receiver buffer requirements and throughput under a heterogeneous wireless environment.

Hence, our contributions are two-fold: (a) Unlike recently proposed CSD schedulers that exploit only the instantaneous behavior of the wireless channel, our scheduler introduces the novel concept of exploiting the long-term behavior as well and (b) Contrary to prior work on QoS analysis that focused on first-order metrics such as throughput and delay, our analysis allows the computation of second-order metrics, which are essential for the evaluation of the wireless receiver buffer requirement.

The rest of the paper is organized as follows: In Section 2, we define our scheduling problem by specifying the input-traffic and wireless channel models and defining the channel-heterogeneous scheduling scenario. In Section 3, we define our proposed scheduler which is analyzed in Section 4. Numerical results that illustrate the trade-off between buffer requirement and throughput amongst various schedulers are presented in Section 5. Concluding remarks are presented in Section 6.

1.3 Notations

For simplicity of notations, for any discrete variable x_i^j, the superscript j and subscript i always correspond to the *flow* and *slot* indices respectively. However, we reserve the symbol p for probability-related notations, where $p^{\mathcal{E}}$ is the probability of occurrence of event $\mathcal{E}$ and $p_x(X)$ is the probability density function (pdf) of x. We use $E[x]$ and $Var[x]$ to denote the mean and variance of x respectively.

In addition, we denote the vectors $\underline{x}^j$ and $\underline{x}_i$ as comprising the elements $\{x_i^j\}_{i=1}^I$ and $\{x_i^j\}_{j=1}^K$ respectively, where *I* is a relevant space spanned by *i*.

2 Scheduling Problem

In this section, we define and model the input-traffic and wireless channel characteristics for the downlink scheduling problem as depicted in the RHS of Fig. 1. Given these models, our objective is to design a wireless scheduler that achieves a good trade-off amongst various performance metrics for a channel-heterogeneous scheduling scenario.

2.1 Input-traffic Model

Packets (assumed to be fixed size) arriving at the access point are queued into K input-flows, where flow *j* comprises packets destined for wireless receiver *j*. The wireless scheduler allocates fixed-size time slots corresponding to the transmission time of one packet to each flow *j* according to its priority parameter, r^j. If $R = \sum_{j=1}^K r^j$, then $\frac{r^j}{R}$ is the fraction of slots that should be allocated to flow *j* over a given interval.

2.2 Wireless Channel Model

Since the performance of a wireless scheduler is influenced by the channel characteristics, it is pertinent to define the channel model considered in our study. A typical channel model that captures the characteristics defined in Property 1 is the Gilbert-Elliott channel [20], where the channel state $c_i^j \in \{0,\ 1\}$ behaves according to a stationary Two-State Markov Chain (2SMC). The wireless receivers are assumed to be sufficiently separated spatially (e.g., in a Wide-Area Network) such that the channel states of different flows are independent.

We specify the channel model in terms of $\{p_{c^j}(0), g^j, \gamma^j\}_{j=1}^K$, which are defined as follows:

γ^j **:** When c_i^j=1 (*bad* channel), any attempted transmission by flow *j* in slot *i* always fails; on the other hand, when c_i^j=0 (*good* channel), the corresponding probability of a successful transmission is 1-γ^j. We assume that $\gamma^j = \gamma$, 1$\leq j \leq K$, in this study.

$p_{c^j}(0)$ **:** $p_{c^j}(0)$ denotes the steady-state probability of the channel of flow *j* being in state 0 and is an indication of the quality of the channel. It varies according to the distance of wireless receiver *j* from the AP. We assume that the channel quality of all flows are identical, i.e., $p_{c^j}(0)$=$p_c(0)$.

g^j **:** g^j indicates the level of a*g*ility of the error behavior across successive slots for flow *j*, and varies according to the mobility of wireless receiver *j* as well as its environment. For small ϵ, we can categorize the channel according to g^j as follows:

$$g^j = \begin{cases} \epsilon, & \text{Persistent channel;} \\ 1, & \text{Uncorrelated channel;} \\ 2-\epsilon, & \text{Oscillatory channel.} \end{cases}$$

We define the decimal equivalent of the binary sequence $c_i^K c_i^{K-1} \cdots c_i^1$ (denoted by $\ddot{c}_i^K$) as the *ensemble* channel state variable, with state space given by $\{0, 1, 2, \cdots 2^K - 1\}$. Therefore, the corresponding state-transition probability matrix, $\underline{\underline{p}}_{\ddot{c}^K}$, is of dimensions $2^K \times 2^K$ and can be computed, for $K \geq 2$, using the following recurrence relation:

$$\underline{\underline{p}}_{\ddot{c}^K} = \begin{bmatrix} \underline{\underline{p}}_{\ddot{c}^K} \cdot p_{c^K}(0|0) & \underline{\underline{p}}_{\ddot{c}^K} \cdot p_{c^K}(1|0) \\ \underline{\underline{p}}_{\ddot{c}^K} \cdot p_{c^K}(0|1) & \underline{\underline{p}}_{\ddot{c}^K} \cdot p_{c^K}(1|1) \end{bmatrix} \quad (1)$$

where

$$\underline{\underline{p}}_{\ddot{c}^1} = \begin{bmatrix} p_{c^1}(0|0) & p_{c^1}(1|0) \\ p_{c^1}(0|1) & p_{c^1}(1|1) \end{bmatrix}$$

and $p_{c^j}(x|y)$ is the transition probability of c^j from state *y* to state *x*, which can be expressed in terms of ($p_{c^j}(0)$,g^j) as follows:

$$\begin{aligned} p_{c^j}(0|1) &= p_{c^j}(0) \cdot g^j \\ p_{c^j}(1|0) &= (1 - p_{c^j}(0))g^j \end{aligned}$$

If we define $\underline{p}_{\ddot{c}_i^K}$ = $[p_{\ddot{c}_i^K}(C)]_{C=0}^{2^K-1}$, then, for any N>0, we have:

$$\underline{p}_{\ddot{c}_{i+N}^K} = \underline{p}_{\ddot{c}_i^K} \times \prod_{u=1}^{N} \underline{\underline{p}}_{\ddot{c}^K} \quad (2)$$

2.3 A Wireless Scheduler for Channel-Heterogeneous Scenario

For optimal performance, the design of a wireless scheduler must consider both the input characteristics (e.g., packet arrival statistics and r^j) as well as the channel parameters ($p_c(0)$,g^j) of each flow *j*. Our focus is to study the influence of the channel on the scheduler design. Hence, the effects of the input characteristics can be isolated by assuming (a) continuously backlogged input flows (thus, eradicating the effects of arrival statistics) and (b) input-homogeneity i.e., r^j = *r* = 1, 1$\leq j \leq K$.

We consider a channel-*heterogeneous* scheduling scenario given as follows, where $\epsilon \approx 0$:

$$g^j = \begin{cases} \epsilon, & 1 \leq j \leq \eta \ (\mathbf{C}^1); \\ 1.0, & \eta + 1 \leq j \leq K \ (\mathbf{C}^2). \end{cases} \quad (3)$$

For the above scenario, our objective is to design a wireless scheduler that achieves a good trade-off amongst the following performance metrics:

2.3.1 Overall Throughput (*T*)

Let n^j denote the Head-of-Line (HOL) packet delay of flow *j*. We define the throughput of flow *j*, T^j, to be the expected number of packets of flow *j* transmitted successfully in each slot. Due to the assumption of continuous backlog in each input-flow, T^j is related to n^j as follows:

$$T^j = \frac{1}{E[n^j]}$$

Since wireless bandwidth is a scarce resource, it is desirable to maximize the overall throughput, *T*, where

$$T = \sum_{j=1}^{K} T^j = \sum_{j=1}^{K} \frac{1}{E[n^j]} \quad (4)$$

2.3.2 Wireless Receiver Buffer Requirement (*b*)

Let us consider a voice-over-WLAN application. A jitter buffer is typically used at each wireless receiver to smooth the playback of the voice call when there is variation in the arrival time of voice packets. Buffer overflow can occur whenever packet arrivals are excessive, and the resulting packet losses create gaps in the voice communication, which can result in clicks, muting or unintelligible speech. Hence, the design of the wireless receiver in terms of its buffer requirement is important.

While much research focuses on the energy consumption of wireless schedulers (e.g., [21, 22, 23]), we consider the impact of the QoS performance of the wireless scheduler on the buffer requirement of the wireless receiver to achieve acceptable voice quality.

Under high load conditions and assuming zero propagation delay in the wireless media, the minimum buffer size, b^j_{min}, to sustain a packet dropping rate, β, for flow *j* can be approximated as follows [24]:

$$b^j_{min} \approx \frac{\left\lceil \frac{\ln \beta}{\ln[Var[n^j]-2E[n^j](1-\rho)]-\ln Var[n^j]} - 1 \right\rceil}{\rho \cdot E[n^j]}$$

where ρ is the utilization factor at the wireless receiver and $\lceil y \rceil$ denotes the smallest integer greater than or equal to *y*. For a given $E[n^j]$, we note that b^j_{min} increases with $Var[n^j]$, and hence, it is desirable for the wireless scheduler to have a small HOL packet delay variation.

3 A hybrid Channel-State Dependent / Fair-Aggregation Scheduler for Heterogeneous Channels

In our prior work [25], we considered a special case of a channel-*homogeneous* scenario, i.e., η = K in Eq. (3). In that work, a stochastic analysis of a CSD scheduler (see Section 3.1) was performed and the stationary HOL packet delay pdf derived, from which various useful performance metrics are obtained. We also introduced a Fair-Aggregation (FA) Scheduler, which simply dispatches packets from each input flow in a round robin manner into a single queue before transmission into the wireless media in a FIFO manner. Based on numerical results, it was deduced that while the FA scheduler achieves better QoS performance when the channel is uncorrelated, the CSD scheduler is superior when the channel is persistent.

Hence, for the scenario defined by Eq. (3), we propose a novel hybrid scheduler that achieves the relative merits of CSD and FA scheduling by partitioning the input flows into ($\mathbf{C}^1$,$\mathbf{C}^2$) according to g^j and applying the respective scheduling mechanism to each group. We denote such a hybrid scheduler as a (*K*,η) CSD-FA scheduler, whose architecture is shown in the LHS of Fig. 2.

We note that the (*K*,η) CSD-FA scheduler is in fact a generalization of the *K*-flow CSD scheduler and a *K*-flow FA scheduler; a (*K*,*K*) CSD-FA scheduler is equivalent to a *K*-flow CSD scheduler while a (*K*,0) CSD-FA scheduler corresponds to a *K*-flow FA scheduler. The mechanism of the hybrid scheduler will be described in the next section.

3.1 Mechanism of (*K*,η) CSD-FA Scheduler

The mechanism of the scheduler can be described in two stages (refer to LHS of Fig. 2). In the first stage, the scheduler dispatches packets from flows in $\mathbf{C}^2$ in a round robin manner into a single queue. If we denote this queue by η', then the second stage comprises a η+1-flow CSD scheduler (with flow composition given by $\mathbf{C}^1 \cup \eta'$), where $\underline{r}$ = [1,$\cdots$,1,*K*-η].

We consider a CSD scheduler model that is similar to the one defined in [7] and maps to the Unified Wireless-Fair Queueing architecture defined in [18]. It comprises a Slot Allocation Policy (SAP), a Channel Status Monitor (CSM), an Arbitration Scheme (AS) and a Packet Dispatcher (DISP), as depicted in the RHS of Fig. 2.

At the beginning of each slot *i*, the AS assigns a transmission priority to each flow based on the SAP and CSM, and the DISP dispatches the HOL packet of the flow with the highest priority for transmission. We describe the mechanism of each component as follows.

3.1.1 SAP

Under error-free conditions, the mechanism (and hence the performance) of the wireless scheduler is determined by the SAP. We restrict the choice of the SAP to perfectly-fair loop schedulers (denoted by $\mathbf{F}^{\underline{r}}$) as they are simple to implement and are mathematically tractable. They possess the following properties:

Property 2 *If the SAP* $\in \mathbf{F}^{\underline{r}}$ *allocates slot i to flow* a_i^{SAP}, *then*

- *For any i>0,* $a_i^{SAP} = a_{i+R}^{SAP}$;

- *Within any interval of R slots, r^j slots must be allocated to flow j, 1≤ $j \leq K$.*

In this paper, we consider a simple Weighted Round Robin (WRR) SAP, which simply allocates r^1 slots to flow 1 followed by r^2 slots to flow 2 and so on. It is easy to show that this scheduler satisfies Property 2. For simplicity of notations, we drop the superscript SAP in a_i^{SAP}.

3.1.2 CSM

The CSM maintains the history of the ensemble channel state based on feedback (see Section 3.1.4) from wireless receivers on the status of each downlink transmission, and uses this information for channel prediction.

Specifically, at the beginning of each slot *i*, $\ddot{c}_{i-x}^K$, *x*>0 is available and is used to generate the prediction, $\hat{\ddot{c}}_i^K$, of the current channel state, $\ddot{c}_i^K$. We consider a probabilistic one-step predictor (OSP) with parameters ($p_{\hat{0}}$,$p_{\hat{1}}$) defined as follows:

$$\text{Prob}(\hat{c}_i^j = c_{i-1}^j \mid c_{i-1}^j = C) \quad = \quad \begin{cases} p_{\hat{0}}, & C = 0; \\ p_{\hat{1}}, & C = 1. \end{cases} \tag{5}$$

The predictor parameters ($p_{\hat{0}}$,$p_{\hat{1}}$) are typically close to 1 since most channels are bursty in nature.

3.1.3 AS

The AS attempts to emulate the performance of the SAP under error-prone conditions based on (a_i, $\hat{\ddot{c}}_i^K$). Its mechanism comprises the following:

Eligibility : This component determines which flows are 'eligible' for transmission. In order to maximize channel efficiency, a flow *j* is *eligible* for transmission in slot *i* only if $\hat{c}_i^j$ = 0, since this increases the likelihood of a successful transmission. Hence, if $\mathbf{G}_i$ denotes the set of eligible flows in slot *i*, then:

$$\mathbf{G}_i = \{\arg_{1 \leq m \leq K} \hat{c}_i^m = 0\}$$

Priority Assignment and Selection : This component *assigns* a priority to each eligible flow and *selects* the flow $f_i \in \mathbf{G}_i$ with the highest priority for transmission.

Since the AS emulates the SAP, the highest priority should be assigned to $flowa_i$ if it is eligible; otherwise, the arbitration function, $Arb()$, determines the alternative eligible flow to be transmitted for transmission. Therefore, we have the following:

$$f_i \quad = \quad \begin{cases} a_i, & a_i \in \mathbf{G}_i; \\ Arb(\mathbf{G}_i), & \text{otherwise.} \end{cases} \tag{6}$$

In this paper, we consider a simple uniform arbitration scheme, where all eligible flows have equal priorities to be selected for transmission, i.e.,

$$\text{Prob}(Arb(\mathbf{G}_i) \quad = \quad \begin{cases} \frac{1}{|\mathbf{G}_i|}, & j \in \mathbf{G}_i; \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

3.1.4 DISP

The DISP dispatches the HOL packet of flow f_i for transmission. Under ideal conditions where channel prediction is perfect and γ=0, the transmission will always be successful; however, such conditions do not hold in reality, and hence, packets received erroneously may have to be re-transmitted. The choice of an ARQ mechanism for re-transmission is important since it affects the QoS performance of the wireless scheduler.

In this study, we consider a simple Stop-and-wait ARQ, where a copy of the transmitted packet is stored in a separate buffer in the DISP. The scheduler is notified about the outcome of each transmission through feedback from the wireless receiver, and we assume that all feedbacks are correctly received. With a failed transmission, the packet is enqueued to the HOL of flow f_i for retransmission; otherwise, the copied packet is deleted from the buffer.

3.2 Illustration of Mechanism of (*K*,η) CSD-FA Scheduler

We illustrate the mechanism of our proposed scheduler by considering a (4,2) CSD-FA scheduler that uses a *deterministic* one-step channel predictor, where $p_{\hat{0}}$ = $p_{\hat{1}}$ = 1 in Eq. (5).

According to the Section 3.1, the (4,2) CSD-FA scheduler is equivalent to a 3-flow CSD scheduler with $\underline{r}$ = [1,1,2] and $\underline{g}$=[ϵ,ϵ,1.0], as depicted in the LHS of Fig. 3. According to the WRR allocation policy, the allocation sequence, $\underline{a}$, is given as follows:

$$\underline{a} \quad = \quad [\cdots, 2, 2', 2', 1, 2, 2', 2', 1, \cdots] \tag{8}$$

Let us assume the following initial conditions: a_0=1 and a flow 3 packet is HOL at flow 2' at the end of slot 0. If TX_i denotes the flow index of the packet transmitted in slot *i*, then the evolution of *TX* corresponding to some channel process $\underline{\ddot{c}}^K$ is depicted in the RHS of Fig. 3.

Since a_0=1, according to Eq. (8), a_1=2; similarly, since c_0^2=0, according to Eq. (5), $\hat{c}_1^2$=0. Hence, according to Eq. (6), flow 2 is selected for transmission. However, since c_1^2=1, the transmission is unsuccessful. The next slot is allocated to flow 2'. Since the HOL packet of flow 2' belongs to flow 3 and c_1^3=0, flow 2' is selected for transmission. The transmission is successful since c_2^3=0.

Slot 3 is again allocated to flow 2' according to Eq. (8). However, since its HOL packet belongs to flow 4 and c_2^4=1, $\hat{c}_3^4$=1, and hence its transmission is deferred. Since c_2^1=c_2^2=0, $\hat{c}_3^1$=$\hat{c}_3^2$=0, and according to Eq. (7), flow 1 and 2 are equally likely to be selected for transmission.

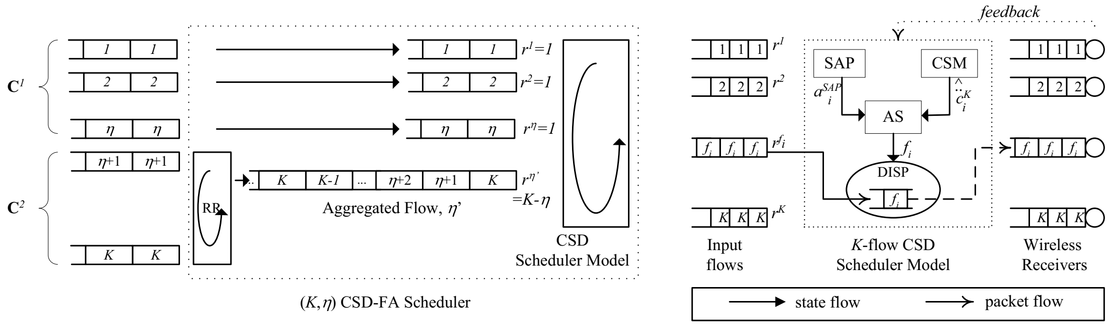

Figure 2: Hybrid CSD-FA Scheduler Model: Flows in $\mathbf{C}^2$ are aggregated into a single flow η'; Flows in $\mathbf{C}^1 \cup \eta'$ are then scheduled by a η+1-flow Channel-State Dependent (CSD) scheduler with $\underline{r}$ = [1,1,$\cdots$ 1,K-η] and $\underline{g}$ = [ϵ,$\cdots$,ϵ,1.0] (left) CSD scheduler model, with illustration of state flow, downlink packet flow (dashed) and uplink packet flow (dotted) in slot i(right).

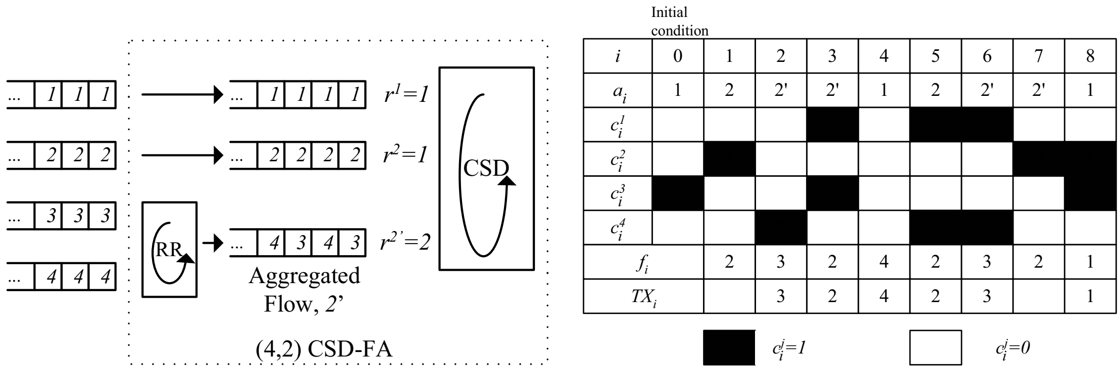

Figure 3: Illustration of the mechanism of a (4,2) CSD-FA scheduler: Architecture (left) and illustration of the mechanism (right) of a (4,2) CSD-FA scheduler with deterministic OSP, AS = UA and SAP = WRR.

We assume that flow 2 is selected, and its transmission is successful since c_3^2=0. Subsequent values of TX can be evaluated in a similar manner.

4 Performance Analysis of (K,η) CSD-FA Scheduler

The performance metrics defined in Section 2.3 can be obtained as long as $p_{n^j}(N^j)$ is given, $1 \le j \le K$. We outline the matrix formulation proposed in our earlier work [19] to evaluate $p_n(N)$ for a K-flow CSD scheduler. We show how this formulation is applied to evaluate $p_{n^j}(N^j)$, $j \in \mathbf{C}^1$. Subsequently, we detail the analysis to derive the corresponding $p_{n^j}(N^j)$ for $j \in \mathbf{C}^2$.

4.1 Notion of Constrained State-Transition Matrix for K-flow CSD Scheduler

Let $\mathcal{S}^j_{a_i}$ ($\mathcal{F}^j_{a_i}$) denote a *S*uccessful (de*F*erred or *F*ailed) transmission of flow j in a slot allocated to flow a_i. The probability of occurrence of $\mathcal{S}^j_{a_i}$ is determined by the AS, the values of ($\ddot{c}^K_{i-1}, \ddot{c}^K_i$) and the SAP. Conversely stated, given the SAP and the AS, the occurrence of $\mathcal{S}^j_{a_i}$ imposes a constraint on $\underline{p}_{\ddot{c}^K_{i-1}}$ and $\underline{p}_{\ddot{c}^K_i}$. Hence, we define the *constrained state-transition matrix* for event $\mathcal{S}^j_{a_i}$ as follows:

$$\underline{p}^{\mathcal{S}^j_{a_i}}_{\ddot{c}^K} \quad = \quad \underline{\underline{D}}^{\mathcal{S}^j_{a_i}}_{i-1}() \times \underline{p}_{\ddot{c}^K} \times \underline{\underline{D}}^{\mathcal{S}^j_{a_i}}_{i}() \qquad (9)$$

where $\underline{\underline{D}}^{\mathcal{S}^j_{a_i}}_x$ is a $2^K \times 2^K$ diagonal matrix such that the diagonal element of row y is the probability that $\mathcal{S}^j_{a_i}$ will occur if $\ddot{c}^K_x$=y-1. Since the events $\mathcal{S}^j_{a_i}$ and $\mathcal{F}^j_{a_i}$ are complementary with respect to flow j,

$$\underline{p}^{\mathcal{S}^j_{a_i}}_{\ddot{c}^K} + \underline{p}^{\mathcal{F}^j_{a_i}}_{\ddot{c}^K} \quad = \quad \underline{p}_{\ddot{c}^K} \qquad (10)$$

Hence, $\underline{p}^{\mathcal{F}^j_{a_i}}_{\ddot{c}^K}$ can be evaluated from $\underline{p}^{\mathcal{S}^j_{a_i}}_{\ddot{c}^K}$ and $\underline{p}_{\ddot{c}^K}$. In a similar manner, we define the *constrained* pdf of $\ddot{c}^K_i$ for event $\mathcal{E}^j_{a_i}$ as follows:

$$\underline{p}^{\mathcal{E}^j_{a_i}}_{\ddot{c}^K_i} \quad = \quad [\text{Prob}(\ddot{c}^K_i = C,\ \mathcal{E}^j_{a_i} \text{ occurs})]^{2^K-1}_{C=0}$$

where $\mathcal{E} \in \{\mathcal{S}, \mathcal{F}\}$. Then, using Eq. (2), we obtain the following *:

$$\underline{p}_{\ddot{c}^K_{i+N}}^{\{\mathcal{E}^j_{a_u}\}_{u=i}^{i+N}} = \underline{p}_{\ddot{c}^K_i}^{\mathcal{E}^j_{a_i}} \times \prod_{u=i+1}^{i+N} \underline{\underline{p}}_{\ddot{c}^K}^{\mathcal{E}^j_{a_u}}$$

from which we have

$$\begin{aligned} p^{\{\mathcal{E}^j_{a_u}\}_{u=i}^{i+N} \mid i} &= \sum_{C=0}^{2^K-1} \underline{p}_{\ddot{c}^K_{i+N}}^{\{\mathcal{E}^j_{a_u}\}_{u=i}^{i+N}} \\ &= \underline{p}_{\ddot{c}^K_i}^{\mathcal{E}^j_{a_i}} \times \prod_{u=i+1}^{i+N} \underline{\underline{p}}_{\ddot{c}^K}^{\mathcal{E}^j_{a_u}} \times \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \end{aligned}$$

Un-conditioning on *i*, we have the following:

$$p^{\{\mathcal{E}^j_{a_u}\}_{u=i}^{i+N}} = \sum_{i=1}^{R} p^{\{\mathcal{E}^j_{a_u}\}_{u=i}^{i+N} \mid i} \tag{11}$$

4.2 Evaluation of $p_n(N)$ for *K*-flow CSD Scheduler

The HOL packet delay for flow *j* is N^j slots when consecutive successful transmissions of flow *j* take place N^j slots apart. Substituting $\{\mathcal{E}^j_{a_u}\}_{u=i}^{i+N^j}$ = $\{\mathcal{S}^j_{a_i}, \{\mathcal{F}^j_{a_{i+u}}\}_{u=1}^{N^j-1}, \mathcal{S}^j_{a_{i+N^j}}\}$ into Eq. (11), we obtain the following expression for $p_{n^j}(N^j)$:

$$\begin{aligned} p_{n^j}(N^j) &= p^{\mathcal{S}^j_{a_i}, \{\mathcal{F}^j_{a_{i+u}}\}_{u=1}^{N^j-1}, \mathcal{S}^j_{a_{i+N^j}}} \\ &= \sum_{i=1}^{R} \underline{p}_{\ddot{c}^K_i}^{\mathcal{S}^j_{a_i}} \times \prod_{u=i+1}^{i+N^j-1} \underline{\underline{p}}_{\ddot{c}^K}^{\mathcal{F}^j_{a_u}} \times \underline{\underline{p}}_{\ddot{c}^K}^{\mathcal{S}^j_{a_{i+N^j}}} \\ &\quad \times \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \end{aligned}$$

Expressions for $\{\underline{p}(S^j_{a_i})\}_{a_i=1}^{R}$, $1 \leq j \leq K$ can be evaluated by a recurrence relation in terms of $\{\underline{\underline{S}}^j_{a_i}\}_{a_i=1}^{R}$, $1 \leq j \leq K$. The evaluation of the latter depends on the predictor parameters, ($p_{\hat{0}}$,$p_{\hat{1}}$), as well as the arbitration function, $Arb()$. Details of these evaluation can be found in [19].

4.3 Evaluation of $p_{n^j}(N^j)$, $j \in \mathbf{C}^1$

We can apply the matrix formulation described in Section 4.1 in the evaluation of $p_{n^j}(N^j)$, $j \in \mathbf{C}^1$, by defining an equivalent η+1-flow CSD scheduling scenario with $\underline{r}$=[1,$\cdots$,1,K-η] and $\underline{g}$=[ϵ,$\cdots$,ϵ,1.0].

In fact, if we define the probabilistic parameters ($p_{\mathcal{S}^x \mid m}$,$p_{\mathcal{D}}$) as follows:

$p_{\mathcal{D}}$ $\equiv$ Prob(a flow *d*efers its transmission attempt)

$p_{\mathcal{S}^x \mid m}$ $\equiv$ Prob(a flow $\in \mathbf{C}^x$ transmits *s*uccessfully $\mid m$ other eligible flows exist)

then $E[n^j]$ can be expressed in terms of ($p_{\mathcal{S}^1 \mid 0}$,$p_{\mathcal{D}}$) by the following theorem:

Theorem 1 *For the scheduling scenario defined in Eq. (3), the expected HOL packet delay for flow j* $\in$ **C**[1] *for a (K,η) CSD-FA scheduler is given as follows:*

$$E[n^j] = \frac{K \cdot \eta(1-p_{\mathcal{D}})}{p_{\mathcal{S}^1 \mid 0}[\eta(1-p_{\mathcal{D}}) + (K-1)(p_{\mathcal{D}} - p_{\mathcal{D}}^{\eta+1})]}$$

where ($p_{\mathcal{S}^1 \mid 0}$,$p_{\mathcal{D}}$) can be expressed in terms of ($p_c(0)$,ϵ) and ($p_{\hat{0}}$,$p_{\hat{1}}$) as follows:

$$\begin{aligned} p_{\mathcal{S}^1 \mid 0} &= p_c(0)[p_{\hat{0}}(1-\epsilon+\epsilon \cdot p_c(0)) \\ &+ (1-p_c(0))(1-p_{\hat{1}})\epsilon](1-\gamma) \\ p_{\mathcal{D}} &= p_c(0)(1-p_{\hat{0}}) + (1-p_c(0))p_{\hat{1}} \end{aligned}$$

Details of the proof of Theorem 1 can be found in [26].

4.4 Evaluation of $p_{n^j}(N^j)$, $j \in \mathbf{C}^2$

In order to simplify the analysis, we assume that any flow $j \in \mathbf{C}^2$ is permitted to transmit *only* in slots allocated to $\mathbf{C}^2$. In any slot within such an interval, if flow *j*'s packet is HOL, then the probability that it will transmit successfully is *independent* of the channel states of all other flows, and is given as follows:

$$p_{\mathcal{S}^2} = p_c(0)[p_{\hat{0}}p_c(0) + (1-p_c(0))(1-p_{\hat{1}})](1-\gamma)$$

Let us denote $p_{\overline{\mathcal{S}^2}}$ = 1-$p_{\mathcal{S}^2}$ as the corresponding probability that no successful flow *j* transmission occurs.

Assume that flow *j* transmits in slot *i*, $1 \leq i \leq \kappa$, where κ is the number of flows with uncorrelated channels. From Fig. 4, we note that κ-1 packets, one from every other flow, must be transmitted before the next flow *j* packet transmits in slot *k*, where 1$\leq$ *k* (modulo *K*)$\leq \kappa$. Since there are κ available transmission slots in the interval [*i*+1:*i*+*K*], we have *k*$\geq$ *K*+*i*.

Over the interval [*i*+1:*k*-1], if we write *k*=*x*$\cdot$ *K*+*y*, then there are *x*$\cdot$ κ+*y*-*i*-1 available transmission slots in this interval, out of which κ-1 slots must contain successful transmissions. In addition, since the scheduling scenario is homogeneous with respect to flows in $\mathbf{C}^2$, under steady-state conditions, *i* is uniformly distributed in the interval 1$\leq$ *i*$\leq$ κ. Therefore, we can write the following for *k*$\geq$ *K*+*i* and 1$\leq$ *y*, *i*$\leq$ κ:

$$\text{Prob}(k = x \cdot K + y) = \frac{\binom{x \cdot \kappa + y - i - 1}{\kappa - 1} p_{\mathcal{S}^2}^{\kappa-1} p_{\overline{\mathcal{S}^2}}^{y-i} p_{\mathcal{S}^2}}{\kappa}$$

Since n^j=*k*-*i*, p_{n^j} is obtained for $N^j \geq K$ and 1$\leq$ *y*, *i* $\leq \kappa$ as follows:

$$p_{n^j}(x \cdot K + y - i) = \frac{\binom{\kappa + y - i - 1}{\kappa - 1} p_{\mathcal{S}^2}^{\kappa} p_{\overline{\mathcal{S}^2}}^{y-i}}{\kappa} \tag{12}$$

Using Eq. (12), we obtain an expression for $E[n^j]$ in the following theorem:

*Note that the notation $\prod_a^b$ refers to a sequence of matrix products in the order *a*,*a*+1,*a*+2,$\cdots$ *b*.

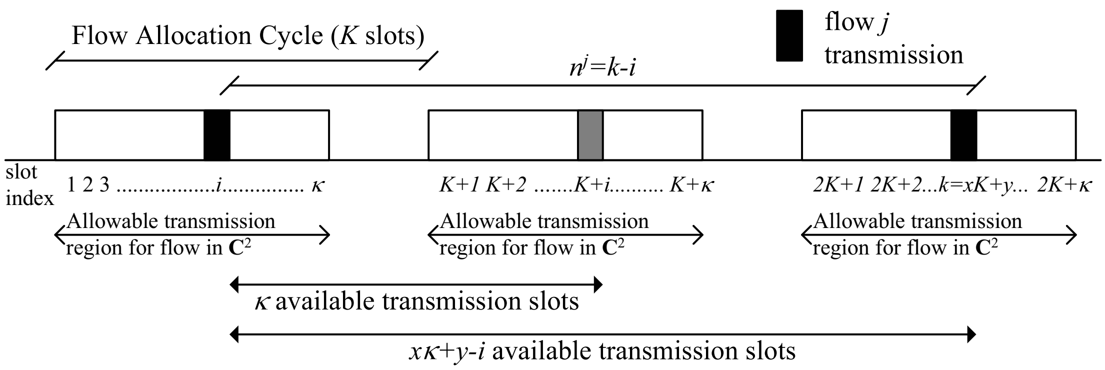

Figure 4: Illustration of the evaluation of $p_{n^j}(N^j)$ for each flow $j \in \mathbf{C}^2$.

Theorem 2 *For the scheduling scenario defined in Eq. (3), the expected HOL packet delay for flow j* $\in \mathbf{C}^2$ *for a (K,η) CSD-FA scheduler is given as follows:*

$$E[n^j] = \frac{K}{p_{\mathbb{S}^2}}$$

Details of the proof of Theorem 2 can be found in [26].

5 Numerical Results

In this section, we compare the overall throughput as well as the wireless receiver buffer requirement between a (*K*,η) *CSD*-FA scheduler and a *K*-flow CSD scheduler for the scheduling scenario given by Eq. (3). We denote the metric *x* corresponding to scheduler π by x_π. We assume a deterministic OSP for channel prediction and γ =0.

Substituting Theorem 1 and 2 into Eq. (4), T_{CSD-FA} can be evaluated and is given in Eq. (13). The corresponding expression for T_{CSD} can be evaluated [26] and is given in Eq. (14).

For a channel-heterogenous scenario, $b^j \neq b^k$ for $j \neq k$. Therefore, we evaluate the average buffer requirement of scheduler π, $\check{b}_\pi$, defined as follows:

$$\check{b}_\pi = \frac{1}{K}\sum_{j=1}^{K} b_\pi^j$$

5.1 Comparison of Throughput and Buffer Requirement of CSD-FA and CSD scheduler

For a given *K*, the metrics *T* and $\check{b}$ depend on the flow composition, η, as well as the channel parameters, $p_c(0)$ and ϵ. We illustrate the effects of each parameter on *T* and $\check{b}$ for *K* = 7, β = 0.01 and ρ = 0.99.

5.1.1 Effects of flow composition

We consider the variation of $\check{b}$ and *T* with η for $p_c(0)$=0.9 and ϵ=0.1 in Fig. 5. As the composition of flows with persistent channels (i.e., η) is increased, *T* is increased since the accuracy of channel prediction is better for persistent channels. This reduces the likelihood of a wasted slot due to erroneous prediction. Compared to T_{CSD}, the throughput degradation due to flow-aggregation is relatively invariant with η and is within 2 %.

Since flows with uncorrelated channels have lower delay variation (according to numerical results presented in [27]), the average buffer requirement is increased as the proportion of flows with persistent channels is increased. However, $\check{b}_{CSD-FA} \leq \check{b}_{CSD}$ due to flow-aggregation, and the resultant reduction in buffer requirement is significant (up to 75%) for small values of η.

We note that when η = *K*-1, both schedulers are identical, and hence they should achieve the same performance in terms of buffer requirement and overall throughput. However, the discrepancy in Fig. 5 is due to the assumption made in Section 4.4, which results in a conservative approximation for the overall throughput for the CSD-FA scheduler.

5.1.2 Effects of channel quality

Next, we consider the variation of $\check{b}$ and *T* with $p_c(0)$ for η=3 and ϵ=0.1 in Fig. 6. As the channel quality is improved (i.e., $p_c(0)$ is increased), *T* is increased since more transmission attempts will occur for a given interval of slots and the proportion of slots with successful transmissions will be increased. We note that flow-aggregation actually results in a slight gain in throughput compared to the CSD scheduler when the channel quality is poor ($p_c(0)$ <0.7). This trend is reversed when channel conditions improve. However, the difference in throughput performance between both schedulers is marginal (within 2 %).

The buffer requirement is reduced as the channel quality is improved, since delay variation is reduced as flows are more likely to transmit in slots allocated to them. The reduction in buffer requirement as a result of flow-aggregation is significant (up to 30%).

$$T_{CSD-FA} = \sum_{j=1}^{K} \frac{1}{E[n^j]} \tag{13}$$
$$= \frac{\eta \cdot p_{\mathcal{S}^1 \mid 0}[\eta(1-p_{\mathcal{D}}) + (K-1)(p_{\mathcal{D}} - p_{\mathcal{D}}^{\eta+1})]}{K \cdot \eta(1-p_{\mathcal{D}})} + \frac{(K-\eta) \cdot p_{\mathcal{S}^2}}{K}$$
$$= \frac{1}{K}\left[\frac{p_{\mathcal{S}^1 \mid 0}[\eta(1-p_{\mathcal{D}}) + (K-1)(p_{\mathcal{D}} - p_{\mathcal{D}}^{\eta+1})]}{(1-p_{\mathcal{D}})} + (K-\eta)p_{\mathcal{S}^2}\right]$$

$$T_{CSD} = \frac{1-p_{\mathcal{D}}^K}{(1-p_{\mathcal{D}})K}[\eta \cdot p_{\mathcal{S}^1 \mid 0} + (K-\eta)p_{\mathcal{S}^2}] \tag{14}$$

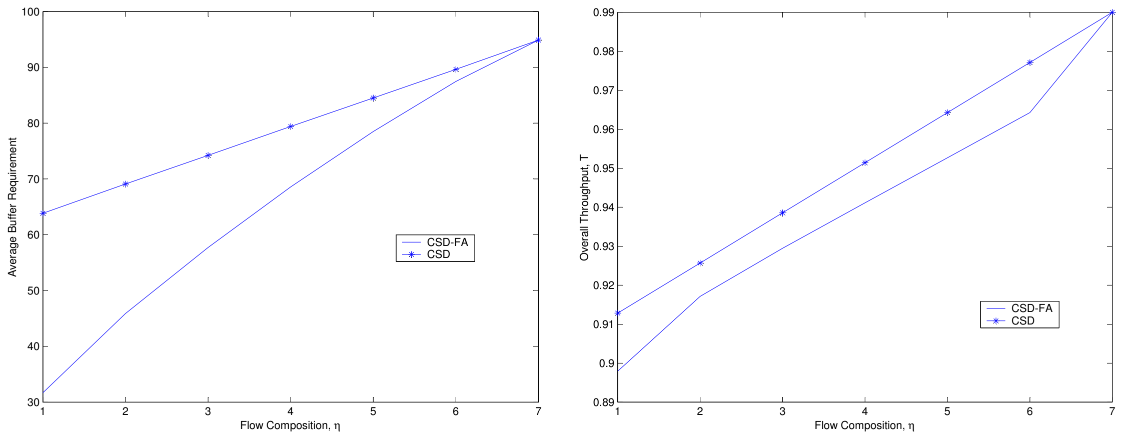

Figure 5: Effects of flow composition on average buffer requirement (left) and overall throughput (right) of CSD schedulers for $p_c(0)$=0.9 and ϵ = 0.1.

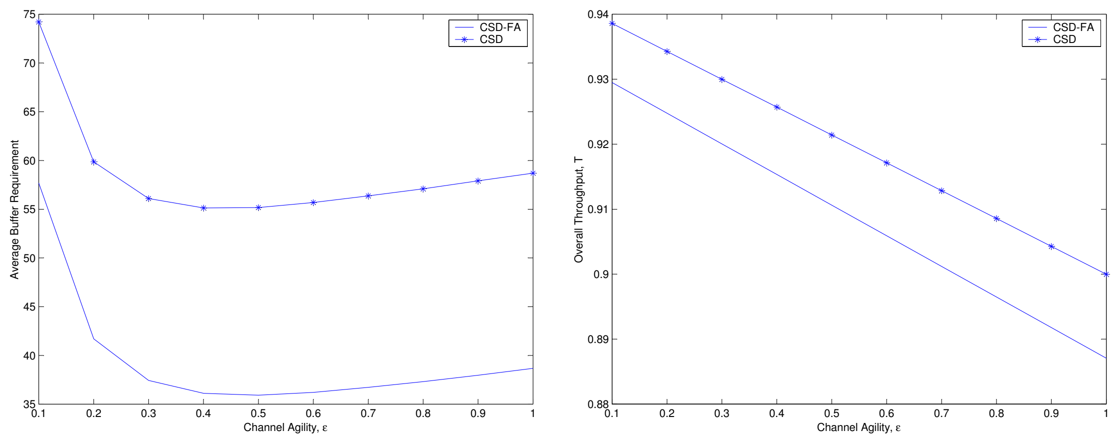

Figure 6: Effects of channel quality on average buffer requirement (left) and overall throughput (right) of CSD schedulers for η=3 and ϵ = 0.1.

5.1.3 Effects of channel burstiness

Lastly, we consider the variation of $\check{b}$ and T with ϵ for $p_c(0)$=0.9 and η=3 in Fig. 7. As the channel for $\mathbf{C}^1$ flows becomes less persistent (i.e., ϵ is increased), T is reduced since the accuracy of channel prediction is reduced. This increases the likelihood of a wasted slot due to erroneous prediction. Compared to T_{CSD}, the throughput degradation due to flow-aggregation is relatively invariant with ϵ and is within 2 %.

Since the buffer requirement of any flow $\in \mathbf{C}^2$ is independent of ϵ, the metric $\check{b}_{CSD-FA}$ is determined by the variation of the buffer requirement of flows $\in \mathbf{C}^1$. It is interesting to note that $\check{b}$ for both schedulers is reduced initially as ϵ is increased, but is increased with further increase in ϵ. However, $\check{b}_{CSD-FA} \leq \check{b}_{CSD}$, and the reduction in buffer requirement is significant (up to 45 %).

5.2 Discussion

A common observation from Section 5.1 is a trade-off between throughput and buffer requirements between the CSD-FA and CSD scheduler: The CSD-FA scheduler results in a significant reduction in the wireless receiver buffer requirement at the expense of reduced throughput compared to the CSD scheduler. In fact, since the throughput degradation is marginal compared to the reduction in buffer requirement, the CSD-FA scheduler is effective in maintaining good overall performance.

Our current analysis assumes a simplistic WRR scheduler as the SAP. However, in [28], we study the performance of various loop schedulers in terms of its delay variation and our analysis indicate that the WRR scheduler exhibits the worst-case performance over the entire class of loop schedulers. Hence, the performance of the CSD-FA scheduler can be enhanced by considering other loop schedulers for the SAP. Several arbitration schemes are proposed in [19] which may result in performance enhancement over uniform arbitration, which is assumed in our study.

Our analysis in Section 4.4 assumes that each flow $j \in \mathbf{C}^2$ is permitted to transmit only in slots allocated to the aggregate flow η'. As a result, the overall throughput computed using Eq. (13) is actually a lower bound to the actual achievable throughput, since slots allocated to $\mathbf{C}^1$ are actually available to flows $\in \mathbf{C}^2$. The corresponding buffer requirement computed represents a lower bound since the delay variation of any flow $j \in \mathbf{C}^2$ is minimized as a result of the assumption.

6 Conclusions

In this paper, we consider the scheduling problem where data packets from K input flows need to be delivered to K corresponding wireless receivers via a heterogeneous wireless channel. Our objective is to design a wireless scheduler that optimizes the buffer requirement at each wireless receiver while maintaining good throughput performance.

We propose a hybrid scheduler that exploits both the short- and long-term error behavior of the channel of each flow so as to achieve high overall throughput as well as low receiver buffer requirements. The scheduler first partitions the flows according to their long-term error behavior (persistent/uncorrelated) such that flows with uncorrelated channels are fairly aggregated. The aggregated flow is then scheduled alongside the remaining flows with a channel-state dependent scheduler, that exploits the short-term error behavior to maximize channel efficiency.

We compare the overall throughput as well as receiver buffer requirements of our proposed scheduler and a channel-state dependent scheduler. Our proposed scheduler achieves good overall throughput as well as low receiver buffer requirements, thus stressing the importance to exploit the long-term error behavior in addition to the instantaneous channel state in the design of wireless schedulers. These parameters can be evaluated using a measurement-based algorithm proposed in [29].

References

[1] A. Demers, S. Keshev, and S. Shenker, "Analysis and simulation of a Fair Queueing Algorithm," *Journal of Internetworking: Research and Experiance*, vol. 1, pp. 3–26, October 1990.

[2] A. K. Parekh and R. G. Gallager, "A Generalized Processor Sharing Approach to Flow Control in Integrated Services Networks - the Single Node Case," *IEEE/ACM Transanctions on Networking*, vol. 1, no. 3, pp. 344–357, June 1993.

[3] H. Sariowan and R. L. Cruz, "SCED: A Generalized Scheduling Policy for Guaranteeing Quality-of-Service," *IEEE/ACM Transanctions on Networking*, vol. 7, no. 5, pp. 669–684, October 1999.

[4] M. Zorzi and R. Rao, "ARQ Error Control for Delay-Constrained Communications on Short-Range Burst-Error Channels," *Proc. of the IEEE VTC*, pp. 1528–1532, May 1997.

[5] K. Lee, "Performance Bounds in Communication Networks with Variable-Rate Links," *Proc. of the ACM SIGCOMM*, pp. 126–136, August 1995.

[6] L. Tassiulas and A. Ephremides, "Dynamic Server Allocation to Parallel Queues with Randomly Varying Connectivity," *IEEE Trans. Information Theory*, vol. 39, no. 2, pp. 466–478, March 1993.

[7] P. Bhagwat, P. Bhattacharya, A. Krishna, and S. Tripathi, "Enhancing throughput over wireless LANs using Channel State Dependent Packet Scheduling," *Proc. of the IEEE INFOCOM*, vol. 3, pp. 1133–1140, March 1996.

[8] X. Liu, Edwin K.P. Chan, and Ness B. Shroff, "Opportunistic Transmission Scheduling with Resource-Sharing Constraints in Wireless Networks," *IEEE Jounnal on Selected Areas in Communications*, vol. 19, no. 10, pp. 2053–2064, October 2001.

[9] S. Shakkottai and A. L. Stolyar, "Scheduling for Multiple Flows Sharing a Time-Varying Channel: The Exponential Rule," *American Mathematical Society Translations Series 2*, vol. 207, pp. 185–202, 2002.

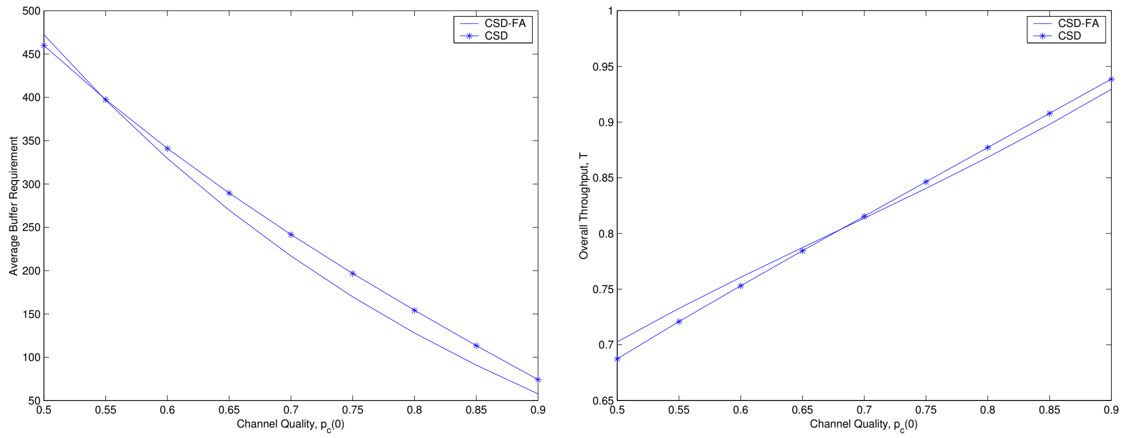

Figure 7: Effects of channel agility on average buffer requirement (left) and overall throughput (right) of CSD schedulers for η=3 and $p_c(0)$ = 0.9.

[10] S. Shakkottai and A. L. Stolyar, "Scheduling Algorithms for a Mixture of Real and Non-Real Time Data in HDR," *Proc. of the 17th International Tele-Traffic Congress*, September 2001.

[11] Y. Cao and V. Li, "Scheduling Algorithms in Broadband Wireless Networks," *Proc. of the IEEE*, vol. 89, no. 1, pp. 76–87, January 2001.

[12] S. Lu, V. Bharghavan, and R. Srikant, "Fair scheduling in wireless packet networks," *Proc. of the ACM SIGCOMM*, pp. 63–74, August 1997.

[13] S. Lu, T. Nandagopal, and V. Bharghavan, "A wireless fair service algorithm for packet cellular networks," *Proc. of the ACM MOBICOM*, pp. 10–20, October 1998.

[14] T.S. Ng, I. Stoica, and H. Zhang, "Packet fair queuing algorithms for wireless networks with location-dependent errors," *Proc. of the IEEE INFOCOM*, vol. 3, pp. 1103–1111, March 1998.

[15] D. Eckhardt and P. Steenkiste, "Effort-limited Fair (ELF) Scheduling for Wireless Networks," *Proc. of the IEEE INFOCOM*, vol. 3, pp. 1097–1106, March 2000.

[16] P. Ramanathan and P. Agrawal, "Adapting Packet Fair Queueing Algorithms to Wireless Networks," *Proc. of the ACM MOBICOM*, pp. 1–9, October 1998.

[17] C. Fragouli, M. Srivastava, and V. Sivaraman, "Controlled Multimedia Wireless Link Sharing via Enhanced Class-based Queuing with Channel-State-Dependent Packet Scheduling," *Proc. of the IEEE INFOCOM*, vol. 2, pp. 572–580, March 1998.

[18] T. Nandagopal, S. Lu, and V. Bharghavan, "A Unified Architecture for the Design and Evaluation of Wireless Fair Queuing Algorithms," *Proc. of the ACM MOBICOM*, pp. 132–142, October 1999.

[19] R. Rom and H. P. Tan, "Framework for Performance Analysis of Channel-aware Wireless Schedulers," *Submitted to Elsevier Journal on Performance Evaluation*, April 2004, Presented in-part at the 23^{rd} IEEE IPCCC, April 2004.

[20] J. R. Yee and E. J. Weldon, "Evaluation of the performance of error-correcting codes on a Gilbert channel," *IEEE Trans. Commun.*, vol. 43, no. 8, pp. 2316–2323, August 1995.

[21] P.J.M. Havinga and G.J.M. Smit, "QoS scheduling for energy efficient wireless communications," *International Conference on Information Technology: Recent Advances in Wireless Communications*, pp. 167–171, April 2001.

[22] P. Nuggehalli, V. Srinivasan, and R. R. Rao, "Delay Constrained Energy Efficient Transmission Strategies for Wireless Devices," *Proc. of the IEEE INFOCOM*, vol. 3, pp. 1765–1772, June 2002.

[23] E. U. Biyikoglu, B. Prabhakar, and A. E. Gamal, "Energy-efficient Scheduling of Packet Transmissions over Wireless Networks," *IEEE/ACM Transactions on Networking*, vol. 10, no. 4, pp. 487–499, August 2002.

[24] R. Rom and H. P. Tan, "Framework for Delay Analysis of Channel-aware Wireless Schedulers," CCIT Tech Report 423, Technion, Israel Institute of Technology, May 2003, Available at `http://www.ee.technion.ac.il/CCIT/info/Publications/Sceintific_e.asp`.

[25] R. Rom and H. P. Tan, "Performance Tradeoffs in Wireless Scheduling with Flow Aggregation," *Proc. of the IEEE WCNC*, vol. 3, pp. 1633–1638, March 2003.

[26] R. Rom and H. P. Tan, "Analysis of Trade-offs between Buffer and QoS Requirements in Wireless Networks," CCIT Tech Report 471, Technion, Israel Institute of Technology, February 2004, Available at http://www.ee.technion.ac.il/CCIT/info/Publications/Sceintific_e.asp.

[27] R. Rom and H. P. Tan, "Stochastic Analysis and Performance Evaluation of Wireless Schedulers," *Wiley Journal of Wireless Communications and Mobile Computing Special Issue: Performance Evaluation of Wireless Networks*, vol. 4, no. 1, pp. 19–41, February 2004.

[28] R. Rom, M. Sidi, and H. P. Tan, "Performance Analysis of a Recursive Cyclic Scheduler for Class-based Scheduling," *Accepted for publication in the* 16^{th} *ITC Specialist Seminar on Performance Evaluation of Wireless and Mobile Systems*, August 2004, Also available as CCIT Tech Report 470, Technion, Israel Institute of Technology, February 2004 at http://www.ee.technion.ac.il/CCIT/info/Publications/Sceintific_e.asp.

[29] A. Konrad, B. Zhao, A. Joseph, and R. Ludwig, "A Markov-based Channel Model Algorithm for Wireless Networks," *Proc. of the ACM MSWiM*, pp. 28–36, July 2001.