Analysis of Trade-offs between Buffer and QoS Requirements in Wireless Networks

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Abstract

In this paper, we consider the scheduling problem where data packets from K input flows need to be delivered to K corresponding wireless receivers over a heterogeneous wireless channel. Our objective is to design a wireless scheduler that optimizes the buffer requirement at each wireless receiver while maintaining good throughput performance. This is a challenging problem due to the unique characteristics of the wireless channel.

We propose a novel idea of exploiting both the long-term and short-term error behavior of the wireless channel in the scheduler design. In addition to typical first-order Quality of Service (QoS) metrics such as throughput and delay, our performance analysis of the scheduler permits the evaluation of higher-order metrics, which are needed to evaluate the buffer requirement. We show that the proposed scheduler achieves high overall throughput as well as low buffer requirement when compared to other wireless schedulers that only make use of the instantaneous channel state in a heterogenous channel.

Index Terms

Wireless Scheduling, QoS, Heterogenous Channel, Buffer Requirements.

I. Introduction

We consider the problem where data packets from K input flows need to be delivered to K corresponding wireless receivers via a wireless media. With the huge success of mobile telephony coupled with a phenomenal growth of internet users, one such scenario is depicted in the wireless network in the left hand side of Fig. 1. We consider the downlink scheduling problem at access point B as shown in the right hand side of Fig. 1.

Packets (assumed to be fixed size) arriving at the access point are queued into K input flows, where flow j comprises packets destined for wireless receiver j. The wireless scheduler allocates fixed-size time slots corresponding to the transmission time of one packet to each flow j according to its priority parameter r^j.

The design of the wireless scheduler is an important problem in wireless networking for:

(a) Wireless application development, since it determines the Quality of Service(QoS), such as throughput and delay guarantees, that the network can support, and

(b) Wireless receiver design, since it determines the buffer requirement at each wireless receiver, which is limited due to size and processing power constraints of portable wireless devices.

In addition to the input flow parameter, r^j, while the capacity of a wired link is usually assumed to be constant, the wireless link is characterized by a (a) high channel error rate (b) bursty and time-varying channel capacity and (c) location dependent channel capacity. This makes the design of a wireless scheduler a hard and challenging problem.

A. Related Work

The design of scheduling policies to meet QoS objectives over a wired link is a well-studied problem ([1], [2], [3], to name a few). Since these guarantees no longer hold over a wireless link, attempts were made to incorporate the effects of the channel characteristics into the guarantees. E.g., in [4], the authors studied the delay performance of a simple ARQ error control strategy for communications over a bursty channel for a *single* flow. In [5], the author investigated the characteristics and traffic effects of variable-rate communication servers. However, the scheduling policy considered is not *channel-aware* since the channel is assumed to be location-independent. Channel-awareness is considered in the resource allocation problem in [6], where the authors characterized the stability properties of the system and proposed an optimal allocation policy that

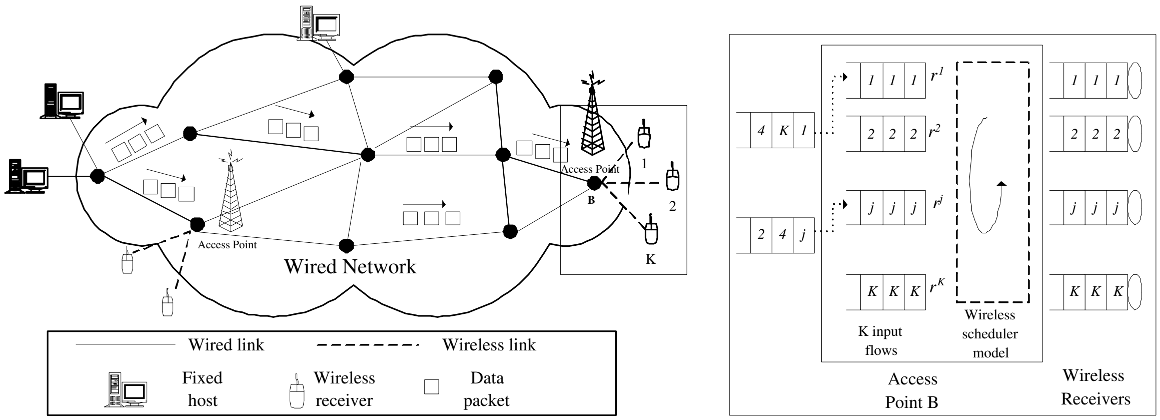


Fig. 1. Wireless scheduling scenario

maximizes throughput and minimizes delay. However, the results apply only when the channel errors are time-uncorrelated.

An alternative approach is to utilize feedback from each receiver to predict the *instantaneous* channel state (i.e., whether it is erroneous or error-free) and the *long-term* behavior such as the burstiness of that channel. Due to characteristics (b) and (c), it is highly likely that at least one receiver with an error-free channel exists at any instant. Hence, in channel-state dependent (CSD) schedulers proposed in [7], [8], by restricting the candidates for transmission to those with *predicted* error-free channels, channel efficiency can be optimized. In [9], [10], the authors considered the downlink scheduling problem in a CDMA system. In this case, the channel information is embedded in the measured data rates, and the authors proposed an exponential rule that optimizes the throughput.

A comprehensive survey of variants of CSD schedulers that differ in the mechanism of selecting the *instantaneous* 'best' flow to transmit while trading-off amongst various performance constraints such as throughput, fairness and delay can be found in [11]. In particular, the concept of 'compensation' was introduced in CSD schedulers proposed in [12], [13], [14], [15], [16], [17] to achieve a tradeoff between channel efficiency and *short-term fairness* provision. These schedulers can be mapped to the unified scheduling architecture proposed in [18]. In addition, the QoS performance of these schedulers in terms of first-order metrics such as throughput and delay are evaluated in this work.

B. Contributions of This Paper

In this paper, we propose a wireless scheduler that partitions the receivers according to the burstiness of its channel, and then applies different scheduling mechanisms to each partition. We present a detailed performance analysis of the proposed scheduler using the framework from our earlier work [19], and show that it achieves a good balance between wireless receiver buffer requirements and throughput under a heterogeneous wireless environment.

Hence, our contributions are two-fold: (a) Unlike recently proposed CSD schedulers that exploit only the instantaneous behavior of the wireless channel, our scheduler introduces the novel concept of exploiting the long-term behavior as well and (b) Contrary to prior work on QoS analysis that focused on first-order metrics such as throughput and delay, our analysis allows the computation of second-order metrics, which are essential for the evaluation of the wireless receiver buffer requirement.

The rest of the paper is organized as follows: In Section II and III, we define the wireless channel and the wireless scheduling problem considered in our study. In Section IV, we define our proposed scheduler which is analyzed in Section V. Numerical results that illustrate the trade-off between buffer requirement and throughput amongst various schedulers are presented in Section VI. Concluding remarks are presented in Section VII.

C. Notations

For simplicity of notations, for any discrete variable x_i^j, the superscript *j* and subscript *i* always correspond to the *flow* and *slot* indices respectively. We denote the vectors $\underline{x}^j$ and $\underline{x}_i$ as comprising the elements $\{x_i^j\}_{i=1}^{I}$ and $\{x_i^j\}_{j=1}^{K}$ respectively, where *I* is a relevant space spanned by *i*. In addition, we use $p_x(X)$, $E[x]$ and $Var[x]$ to denote the probability density function (pdf), mean and variance of *x* respectively.

II. Wireless Channel Model

Since the performance of a wireless scheduler is influenced by the channel characteristics, it is pertinent to define the channel model considered in our study. Let *c* denote the (*instantaneous* or *short-term*) channel state variable. A typical channel model that captures the characteristics (a) to (c) defined in Section I is the Gilbert-Elliott channel [20], where $c_i^j \in \{0,\ 1\}$ behaves according to a Two-State Markov Chain (2SMC). Such a model is usually specified in terms of $(p_{c^j}(1|0), p_{c^j}(0|1))$, where

$$p_{c^j}(x|y) \;=\; \text{Prob}(c_i^j\text{=x} \mid c_{i-1}^j\text{=y})$$

and $p_{c^j}(C)$ is given as follows:

$$p_{c^j}(C) \;=\; \begin{cases} \frac{p_{c^j}(0|1)}{p_{c^j}(1|0)+p_{c^j}(0|1)}, & C=0; \\ \frac{p_{c^j}(1|0)}{p_{c^j}(0|1)+p_{c^j}(1|0)}, & C=1. \end{cases}$$

However, we specify the channel model in terms of ($p_{c^j}(0)$, α^j), where $p_{c^j}(0)$ describes the steady state probability of the channel of flow *j* being in state 0 and $\alpha^j = p_{c^j}(1|0) + p_{c^j}(0|1)$ describes the *long-term* behavior of the channel and indicates the level of *agility* of the error behavior across successive slots for flow *j*. For small ϵ, we can categorize the channel according to α^j as follows:

$$\alpha^j \;=\; \begin{cases} \epsilon, & \text{Persistent channel;} \\ 1, & \text{Uncorrelated channel;} \\ 2-\epsilon, & \text{Oscillatory channel.} \end{cases}$$

We assume that when the channel is in state 0(1), packet transmissions are always (never) successful. In addition, the wireless receivers are sufficiently separated spatially such that the channel state of different flows are independent.

We define the decimal equivalent of the binary sequence $c_i^K c_i^{K-1} \cdots c_i^1$ (denoted by $\bar{c}_i^K$) as the *ensemble* channel state variable, with state space given by $\{0, 1, 2, \cdots 2^K - 1\}$. Therefore, the corresponding state transition probability matrix, $\underline{\underline{C}}^K$, is of dimensions $2^K \times 2^K$ and can be computed, for $K \geq 2$, using the following recurrence relation:

$$\underline{\underline{C}}^K \;=\; \begin{bmatrix} \underline{\underline{C}}^{K-1} \cdot p_{c^K}(0|0) & \underline{\underline{C}}^{K-1} \cdot p_{c^K}(1|0) \\ \underline{\underline{C}}^{K-1} \cdot p_{c^K}(0|1) & \underline{\underline{C}}^{K-1} \cdot p_{c^K}(1|1) \end{bmatrix}$$

where

$$\underline{\underline{C}}^1 \;=\; \begin{bmatrix} p_{c^1}(0|0) & p_{c^1}(1|0) \\ p_{c^1}(0|1) & p_{c^1}(1|1) \end{bmatrix}$$

If we define $\underline{f}_i$ = $[p_{\bar{c}_i^K}(C)]_{C=0}^{2^K-1}$, then, for any N>0, we have:

$$\underline{f}_{i+N} \;=\; \underline{f}_i \times \prod_{u=1}^{N} \underline{\underline{C}}^K \tag{1}$$

III. Scheduling Problem

For optimal performance, the design of a wireless scheduler must consider both the input characteristics (e.g., packet arrival statistics and r^j) as well as the channel parameters ($p_{c^j}(0)$,α^j) of each flow *j*. Our focus is to study the influence of the channel on the scheduler design. Hence, the effects of the input characteristics can be isolated by assuming (a) continuously backlogged input flows (thus, eradicating the effects of arrival statistics) and (b) input homogeneity i.e., r^j = *r* = 1, $1 \leq j \leq K$.

Next, we specify the requirements of the wireless scheduler in terms of performance metrics. We show that these metrics can be computed by evaluating $p_{n^j}(N)$, where n^j denotes the Head-of-Line (HOL) packet delay of flow *j*.

A. Overall Throughput (W)

We define the throughput of flow *j*, W^j, to be the expected number of packets of flow *j* transmitted successfully in each slot. Due to the assumption of continuous backlog in each input flow, W^j is related to n^j as follows:

$$W^j = \frac{1}{E[n^j]}$$

Since wireless bandwidth is a scarce resource, it is desirable to maximize the overall throughput, *W*, where

$$W = \sum_{j=1}^{K} W^j = \sum_{j=1}^{K} \frac{1}{E[n^j]} \tag{2}$$

B. Buffer Size (B) to sustain overflow rate (β)

Since each wireless receiver is limited in terms of buffer size, the wireless scheduler has to maintain an acceptable packet dropping rate due to buffer overflow. Under high load conditions and assuming zero propagation delay in the wireless media, the minimum buffer size, B^j, to sustain a packet dropping rate, β, for flow *j* can be approximated as follows [21]:

$$B^j \approx \frac{\lceil \frac{\ln \beta}{\ln(1-\frac{1}{E[w^j]})} - 1 \rceil}{X}$$

where

$$E[w^j] \;=\; \frac{Var[n^j]}{2E[n^j](1-\rho)}$$

X is the constant wireless receiver service time, ρ = $\frac{X}{E[n^j]}$ and $\lceil y \rceil$ denotes the smallest integer greater than or equal to *y*. For a given $E[n^j]$, we note that B^j increases with $Var[n^j]$, and hence, it is desirable for the wireless scheduler to have a small HOL packet delay variation.

IV. A HYBRID CHANNEL-STATE DEPENDENT / FAIR-AGGREGATION SCHEDULER FOR HETEROGENOUS CHANNELS

In this study, we consider a CSD scheduler model that is similar to the one defined in [7] and maps to the unified wireless scheduling architecture defined in [18]. It comprises a slot allocation policy, a channel status monitor, an arbitration scheme and a packet dispatcher, as depicted in Fig. 2(a).

A. Slot Allocation Policy (SAP)

The SAP allocates each slot *i* to flow a_i to fulfill the performance requirements specified in Section III. Since the allocation is independent of the channel conditions, the performance is only guaranteed under error-free conditions.

We restrict the choice of the SAP to loop schedulers of size *R* (i.e., a_{i+R}=a_i), where *R* = $\sum_{j=1}^{K} r^j$, as they are simple to implement and are mathematically tractable. Specifically, in this paper, we consider a simple Weighted-Round Robin (WRR) allocation policy, which simply allocates r^1 slots to flow 1 followed by r^2 slots to flow 2 and so on.

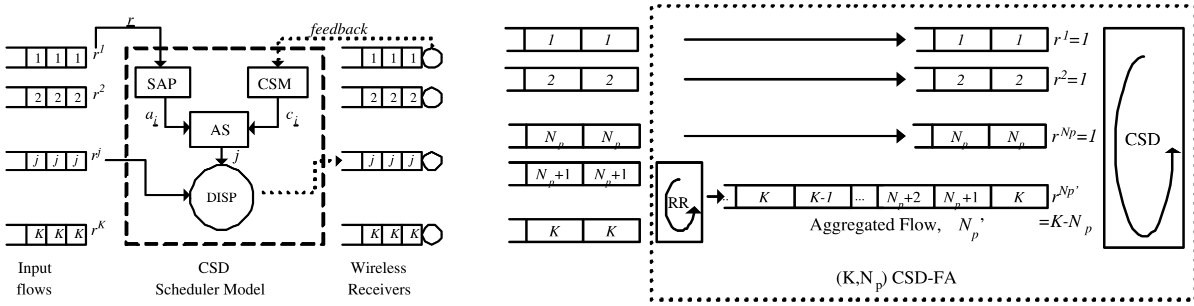
B. Channel Status Monitor (CSM)

We assume that the CSM receives feedback (assumed ideal) from each wireless receiver at the end of each slot. Hence, at the beginning of slot *i*, $\{c^j_{i-m}, m > 0\}_{j=1}^{K}$ is available and is used to generate the prediction, $\hat{\overline{c}}_i^K$, of the current channel state, $\overline{c}_i^K$. In order to maximize channel efficiency, a flow *j* is *eligible* for transmission in slot *i* only if $\hat{c}^j_i$ = 0.

We consider a probabilistic one-step predictor with parameters ($p_{\hat{0}}$,$p_{\hat{1}}$) defined as follows:

$$\text{Prob}(\hat{c}^j_i = c^j_{i-1} \mid c^j_{i-1} = c) \;=\; \begin{cases} p_{\hat{0}}, & c = 0; \\ p_{\hat{1}}, & c = 1. \end{cases} \tag{3}$$

The predictor parameters ($p_{\hat{0}}$,$p_{\hat{1}}$) are typically close to 1 since most channels are bursty in nature.



(a) (b)

Fig. 2. (a) CSD and (b) Hybrid CSD-FA Scheduler Models

C. Arbitration Scheme (AS)

Based on (a_i, $\hat{\bar{c}}_i^K$), the AS determines the *best* eligible flow *j* to transmit in slot *i* according to the following heuristic:

$$j = \begin{cases} a_i, & \hat{c}_i^{a_i} = 0; \\ Arb(\hat{\bar{c}}_i^K), & \text{otherwise}. \end{cases} \tag{4}$$

where $Arb()$ is used to select an alternative eligible flow to transmit when flow a_i is not eligible for transmission. We consider uniform arbitration in this study, where all eligible flows have equal priorities to be selected for transmission, i.e.,

$$\text{Prob}(Arb(\hat{\bar{c}}_i^K) = j) = \begin{cases} \frac{1}{|G|}, & j \in G; \\ 0, & \text{otherwise}. \end{cases} \tag{5}$$

where *G* = $\{\arg_{1 \le m \le K, m \ne a_i} \hat{c}_i^m = 0\}$. Hence, the likelihood that a particular flow will be selected for transmission depends only on its channel condition.

D. Packet Dispatcher (DISP)

The DISP dispatches the HOL packet of flow *j* for transmission, and stores a copy of the packet in a buffer. If the transmission is unsuccessful, it will insert the packet at the HOL of flow *j*.

E. Definition of (K,N_p) CSD-FA Scheduler

In our prior work [22], a *homogeneous* channel was analyzed, i.e., for 1$\le j \le K$,

$$(p_{c^j}(0), \alpha^j) = (p_c(0), \alpha) \tag{6}$$

In that work, a stochastic analysis of the CSD scheduler was performed and the stationary packet delay distribution derived, from which various useful performance metrics are obtained. We also introduced a Fair-Aggregation (FA) Scheduler, which simply dispatches packets from each input flow in a round robin manner into a single queue before transmission into the wireless media in a FIFO manner. Based on numerical results, it was deduced that while the FA scheduler achieves better QoS performance when the channel is uncorrelated, the CSD scheduler is superior when the channel is persistent.

Hence, if the assumption of channel homogeneity in Eq. (6) is relaxed with respect to α^j as follows, where $\epsilon_1 \approx 0$:

$$\alpha^j = \begin{cases} \epsilon_1, & 1 \leq j \leq N_p \ (Gp_1); \\ 1.0, & N_p + 1 \leq j \leq K \ (Gp_2). \end{cases} \tag{7}$$

then we can achieve the relative merits of CSD and FA scheduling by partitioning the input flows into (Gp_1,Gp_2) according to α^j and applying the respective scheduling mechanism to each group. We denote such a hybrid scheduler as a (K,N_p) CSD-FA scheduler.

The mechanism of the scheduler can be described in two stages. In the first stage, the scheduler dispatches packets from flows in Gp_2 in a round robin manner into a single queue. If we denote this queue by N_p', then the second stage comprises a N_p+1-flow CSD scheduler (with flow composition given by $Gp_1 \cup N_p'$), where $\underline{r}$ = [1 $\cdots$,1,K-N_p]. This is illustrated in Fig. 2(b).

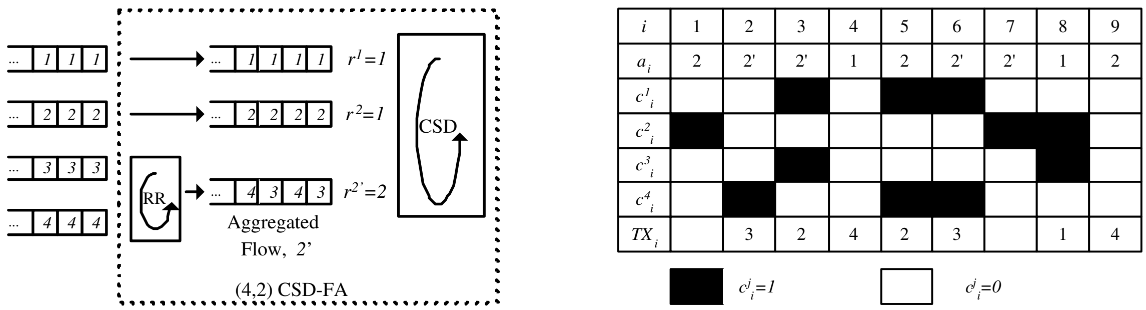
We note that the (K,N_p) CSD-FA scheduler is in fact a generalization of the K-flow CSD scheduler and a K-flow FA scheduler; a (K,K) CSD-FA scheduler is equivalent to a K-flow CSD scheduler while a (K,0) CSD-FA scheduler corresponds to a K-flow FA scheduler.

F. Illustration of Mechanism of (K,N_p) CSD-FA Scheduler

We illustrate the mechanism of our proposed scheduler by considering a (4,2) CSD-FA scheduler that uses a *deterministic* one-step channel predictor, where $p_{\hat{0}}$ = $p_{\hat{1}}$ = 1 in Eq. (3).

According to Section IV-E, the (4,2) CSD-FA scheduler is equivalent to a 3-flow CSD scheduler with $\underline{r}$ = [1 1 2], as depicted in Fig. 3(a). Hence, assuming that a_1=2, the allocation sequence, $\underline{a}$, is given as follows:

$$\underline{a} = [2, 2', 2', 1, 2, 2', 2', 1, \cdots]$$



(a) (b)

Fig. 3. (a) Components and (b) Illustration of the (4,2) CSD-FA Scheduler

In addition, we assume that flow 2' contains a flow 4 packet at its HOL at the beginning of slot 1, and that $\underline{c}_0$=[0 0 1 0]. Let TX_i denote the flow index of the packet transmitted in slot *i*. Then, the evolution of *TX* corresponding to some channel process $\underline{c}$ is depicted in Fig. 3(b).

Since a_1=2 and c_0^2=0, according to Eq. (3), $\hat{c}_1^2$=0. Hence, according to Eq. (4), flow 2 is selected for transmission. However, since c_1^2=1, the transmission is unsuccessful. The next slot is allocated to flow 2'. Since the HOL packet of flow 2' belongs to flow 3 and c_1^3=0, flow 2' is selected for transmission. The transmission is successful since c_2^3=0.

Slot 3 is again allocated to flow 2' according to the WRR policy. However, since its HOL packet belongs to flow 4 and c_2^4=1, $\hat{c}_3^4$=1, and hence its transmission is deferred. Since c_2^1=c_2^2=0, $\hat{c}_3^1$=$\hat{c}_3^2$=0, and according to Eq. (5), flow 1 and 2 are equally likely to be selected for transmission. We assume that flow 2 is selected, and its transmission is successful since c_3^2=0.

V. Performance Analysis of (K,N_p) CSD-FA Scheduler

In Section III, we showed that by computing $p_n(N)$, we can obtain the required performance metrics used to evaluate the wireless scheduler. We begin this section with an outline of the matrix formulation proposed in our earlier work in [19] to evaluate $p_n(N)$ for a general CSD scheduler. We show how this formulation is applied to evaluate $p_{n^f}(N)$, $f \in Gp_1$. Then, we detail the analysis to derive the corresponding $p_{n^f}(N)$ for $f \in Gp_2$.

A. Evaluation of $p_{n^f}(N)$ for K-flow CSD scheduler

Let $S_{a_i}^f$ ($F_{a_i}^f$) denote a **S**uccessful (de**F**erred or **F**ailed) transmission of flow *f* in a slot allocated to flow a_i. The probability of occurrence of $S_{a_i}^f$ is determined by the AS, the values of ($\overline{c}_{i-1}^K, \overline{c}_i^K$)

and *i*. Conversely stated, given *i* and the AS, the occurrence of $S^f_{a_i}$ imposes a constraint on $[p_{\overline{c}^K_{i-1}}(C)]^{2^K-1}_{C=0}$ and $[p_{\overline{c}^K_i}(C)]^{2^K-1}_{C=0}$. Hence, we define the *constrained state transition matrix* for event $S^f_{a_i}$ as follows:

$$\underline{\underline{S}}^f_{a_i} = \underline{\underline{D}}_{i-1}(S^f_{a_i}) \times \underline{\underline{C}}^K \times \underline{\underline{D}}_i(S^f_{a_i}) \tag{8}$$

where $\underline{\underline{D}}_x(S^f_{a_i})$ is a diagonal matrix such that the diagonal element of row *m* is the probability that $S^f_{a_i}$ will occur if $\overline{c}^K_x$=*m*-1. Since the events $S^f_{a_i}$ and $F^f_{a_i}$ are complementary,

$$\underline{\underline{S}}^f_{a_i} + \underline{\underline{F}}^f_{a_i} = \underline{\underline{C}}^K$$

Hence, $\underline{\underline{F}}^f_{a_i}$ can be evaluated from $\underline{\underline{S}}^f_{a_i}$ and $\underline{\underline{C}}^K$.

If we define the *constrained* pdf of the channel state as follows:

$$\underline{f}(E^f_{a_i}) = [\text{Prob}(\overline{c}^K_i = C, E^f_{a_i}\text{occurs})]^{2^K-1}_{C=0}$$

where $E \in \{S, F\}$. Then Eq. (1) can be written as follows *:

$$\underline{f}(\{E^f_{a_u}\}^{i+N}_{u=i}) = \underline{f}(E^f_{a_i}) \times \prod_{u=i+1}^{i+N} \underline{\underline{E}}^f_{a_u}$$

from which we have

$$\begin{aligned}\text{Prob}(\{E^f_{a_u}\}^{i+N}_{u=i} \text{ occurs} \mid i) &= \sum_{C=0}^{2^K-1} \underline{f}(\{E^f_{a_u}\}^{i+N}_{u=i}) \\ &= \underline{f}(E^f_{a_i}) \times \prod_{u=i+1}^{i+N} \underline{\underline{E}}^f_{a_u} \times \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}\end{aligned}$$

Un-conditioning on *i*, we have the following:

$$\text{Prob}(\{E^f_{a_u}\}^{i+N}_{u=i} \text{ occurs}) = \sum_{i=1}^{K} \underline{f}(E^f_{a_i}) \times \prod_{u=i+1}^{i+N} \underline{\underline{E}}^f_{a_u} \times \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

The HOL packet delay for flow *f* is *N* slots when consecutive successful transmissions of flow *f* take place *N* slots apart. In other words, if $\{E^f_{a_u}\}^{i+N}_{u=i} = \{S^f_{a_i}, \{F^f_{a_{i+u}}\}^{N-1}_{u=1}, S^f_{a_{i+N}}\}$, then $p_{n^f}(N)$

*Note that the notation $\prod_a^b$ refers to a sequence of matrix products in the order a,a+1,a+2,$\cdots$ b.

can be evaluated as follows:

$$\begin{aligned} p_{n^f}(N) &= \text{Prob}(S^f_{a_i}, \{F^f_{a_{i+u}}\}_{u=1}^{N-1}, S^f_{a_{i+N}} \text{occurs}) \\ &= \sum_{i=1}^{K} \underline{f}(S^f_{a_i}) \times \prod_{u=i+1}^{i+N-1} \underline{\underline{F}}^f_{a_u} \times \underline{\underline{S}}^f_{a_{i+N}} \times \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \end{aligned}$$

Expressions for $\{\underline{f}(S^f_{a_i})\}^K_{f,a_i=1}$ can be evaluated by a recurrence relation in terms of $\{\underline{\underline{S}}^f_{a_i}\}^K_{f,a_i=1}$. The evaluation of the latter depends on the predictor parameters, ($p_{\hat{0}}$,$p_{\hat{1}}$), as well as the arbitration function, $Arb()$. Details of these evaluation can be found in [19].

B. Evaluation of $p_{n^f}(N)$ for Flow $f \in Gp_1$ of (K,N_p) CSD-FA Scheduler

We can apply the framework described in Section V-A in the evaluation of $p_{n^f}(N)$, $f \in Gp_1$, by defining an equivalent N_p+1-flow CSD scheduling scenario with the following parameters:

$$\begin{aligned} (r^j, \alpha^j) &= \begin{cases} (1, \epsilon_1), & 1 \le j \le N_p; \\ (K - N_p, 1.0), & j = N_p + 1. \end{cases} \\ p_{c^j}(0) &= p_c(0), \quad 1 \le j \le N_p + 1 \end{aligned} \tag{9}$$

Let us define the probabilistic parameters (p_{s_1},p_d) as follows:

$$\begin{aligned} p_{s_i} &\equiv \text{Prob(a flow} \in Gp_i \text{ will transmit successfully)} \\ p_d &\equiv \text{Prob(a flow will defer a transmission attempt)} \end{aligned}$$

Then, $E[n^f]$ can be expressed in terms of (p_{s_1},p_d) according to the following theorem:

Theorem 1: For the channel process defined in Eq. (7), the expected HOL packet delay for flow $f \in Gp_1$ for a (*K*,N_p) CSD-FA scheduler is given as follows:

$$E[n^f] = \frac{K \cdot N_p(1 - p_d)}{p_{s_1}[N_p(1 - p_d) + (K - 1)(p_d - p_d^{N_p+1})]}$$

where (p_{s_1},p_d) can be expressed in terms of ($p_c(0)$,ϵ_1) and ($p_{\hat{0}}$,$p_{\hat{1}}$) as follows:

$$\begin{aligned} p_{s_1} &= p_c(0)[p_{\hat{0}}(1 - \epsilon_1 + \epsilon_1 \cdot p_c(0)) + (1 - p_c(0))(1 - p_{\hat{1}})\epsilon_1] \\ p_d &= p_c(0)(1 - p_{\hat{0}}) + (1 - p_c(0))p_{\hat{1}} \end{aligned} \tag{10}$$

Details of the proof of Theorem 1 can be found in Appendix I.

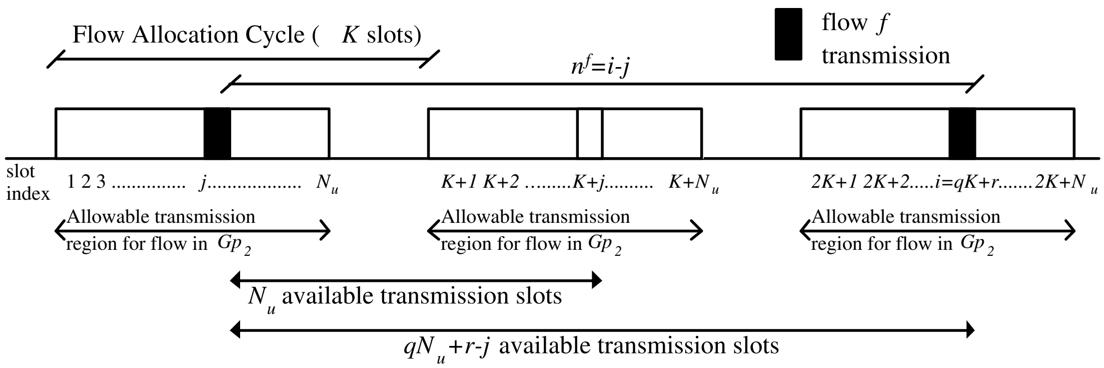


Fig. 4. Illustration of the evaluation of $p_{n^f}(N)$ for each flow $f \in Gp_2$

C. Evaluation of $p_{n^f}(N)$ for Flow $f \in Gp_2$ of (K,N_p) CSD-FA Scheduler

In order to simplify the analysis, we assume that any flow $f \in Gp_2$ *only* transmit in slots allocated to Gp_2. According to Eq. (7), since $\alpha^f = 1.0, \ \forall f \in Gp_2$,

$$p_{s_2} \;\; = \;\; p_c(0)[p_{\hat{0}}p_c(0) + (1 - p_c(0))(1 - p_{\hat{1}})]$$

Let us denote $p_{\overline{s}_2}$ = 1-p_{s_2} as the probability that no successful flow *f* transmission occurs in a given slot, where $f \in Gp_2$.

Assume that flow *f* transmits in slot *j*, $1 \leq j \leq N_u$, where N_u=*K*-N_p is the number of flows with uncorrelated channels. From Fig. 4, we note that N_u-1 packets, one from every other flow, must be transmitted before the next flow *f* packet transmits in slot *i*, where $1 \leq$ *i* (mod *K*)$\leq N_u$. Since there are N_u available transmission slots in the interval [*j*+1:*j*+*K*], we have *i*$\geq$ *K*+*j*. Over the interval [*j*+1:*i*-1], if we write *i*=*q*$\cdot$ *K*+*r*, then there are *q*$\cdot$ N_u+*r*-*j*-1 available transmission slots in this interval, out of which N_u-1 slots must contain successful transmissions. In addition, since the system is homogeneous with respect to flows in Gp_2, under steady-state conditions, *j* is uniformly distributed in the interval $1 \leq$ *j*$\leq N_u$. Therefore, we can write the following for *i*$\geq$ *K*+*j* and $1 \leq$ *r*, *j*$\leq N_u$:

$$\text{Prob}(i = q \cdot K + r) \;\; = \;\; \frac{\binom{q \cdot N_u + r - j - 1}{N_u - 1} p_{s_2}^{N_u - 1} p_{\overline{s}_2}^{r-j} p_{s_2}}{N_u}$$

Since n^f=*i*-*j*, $p_{n^f}(N)$ is obtained for *N*$\geq$ *K* and $1 \leq$ *r*, *j* $\leq N_u$ as follows:

$$p_{n^f}(q \cdot K + r - j) \;\; = \;\; \frac{\binom{N_u + r - j - 1}{N_u - 1} p_{s_2}^{N_u} p_{\overline{s}_2}^{r-j}}{N_u} \tag{11}$$

Using Eq. (11), we obtain an expression for $E[n^f]$ in the following theorem:

Theorem 2: For the channel process defined in Eq. (7), the expected HOL packet delay for flow $f \in Gp_2$ for a (K,N_p) CSD-FA scheduler is given as follows:

$$E[n^f] = \frac{K}{p_{s_2}}$$

Proof: Using Eq. (11), we compute $E[n^f]$ as follows:

$$\begin{aligned} E[n^f] &= \sum_{j=1}^{N_u}(\sum_{q=1}^{1}\sum_{r=j}^{N_u}+\sum_{q=2}^{\infty}\sum_{r=1}^{N_u})[q\cdot K+r-j]\cdot p_{n^f}(q\cdot K+r-j) \\ &= \frac{p_c(0)^{N_u}}{N_u}\sum_{j=1}^{N_u}(\sum_{q=1}^{1}\sum_{r=j}^{N_u}+\sum_{q=2}^{\infty}\sum_{r=1}^{N_u})[q\cdot K+r-j]\binom{N_u+r-j-1}{N_u-1}p_c(1)^{r-j} \end{aligned}$$

We can simplify the above expression by noting that the exponent of the term $p_c(1)$ ranges from 0 to ∞, and by evaluating the coefficient of $\{p_c(1)^w\}_{w=0}^{\infty}$, we obtain the following expression:

$$E[n^f] = \frac{p_c(0)^{N_u}}{N_u}\sum_{w=0}^{\infty}K(N_u+w)\binom{w+N_u-1}{N_u-1}p_c(1)^w \tag{12}$$

From binomial theorem, we have the following result:

$$\sum_{w=0}^{\infty}\binom{w+p}{p}y^w = \frac{1}{(1-y)^{p+1}} \tag{13}$$

Differentiating Eq. (13) with respect to y, we obtain

$$\sum_{w=0}^{\infty}w\binom{w+p}{p}y^{w-1} = \frac{p+1}{(1-y)^{p+2}} \tag{14}$$

Substituting Eq. (13) and Eq. (14) into Eq. (12), we obtain the expression for $E[n^f]$ as given in Theorem 2. ■

VI. Numerical Results

In this section, we compare the overall throughput, *W*, as well as the wireless receiver buffer requirement, *B*, between a (K,N_p) CSD-FA scheduler and a K-flow CSD scheduler for the heterogeneous channel process defined in Eq. (7). We denote the metric *A* corresponding to scheduler π by A_π.

Based on Theorem 1 and 2, we have the following expression for $E[n^f]$ for the (K,N_p) CSD-FA scheduler:

$$E[n^f] = \begin{cases} \frac{K\cdot N_p(1-p_d)}{p_{s_1}[N_p(1-p_d)+(K-1)(p_d-p_d^{N_p+1})]}, & f\in Gp_1; \\ \frac{K}{p_{s_2}}, & f\in Gp_2. \end{cases}$$

Hence, according to Eq. (2), W_{CSD-FA} can be evaluated as follows:

$$\begin{aligned} W_{CSD-FA} &= \sum_{f=1}^{K} \frac{1}{E[n^f]} \\ &= N_p \cdot \frac{p_{s_1}[N_p(1-p_d)+(K-1)(p_d-p_d^{N_p+1})]}{K \cdot N_p(1-p_d)} + (K-N_p) \cdot \frac{p_{s_2}}{K} \\ &= \frac{1}{K}[\frac{p_{s_1}[N_p(1-p_d)+(K-1)(p_d-p_d^{N_p+1})]}{(1-p_d)} + (K-N_p)p_{s_2}] \end{aligned} \quad (15)$$

The corresponding expression for W_{CSD} (Appendix I) is given as follows:

$$W_{CSD} = \frac{1-p_d^K}{(1-p_d)K}[N_p p_{s_1} + (K-N_p)p_{s_2}] \quad (16)$$

Since the channel process is heterogenous, $B^j \neq B^k$ for $j \neq k$. Therefore, we evaluate the buffer requirement of scheduler π in terms of its *average* buffer requirement, B_π^{avg}, defined as follows:

$$B_\pi^{avg} = \frac{1}{K}\sum_{f=1}^{K} B_\pi^f$$

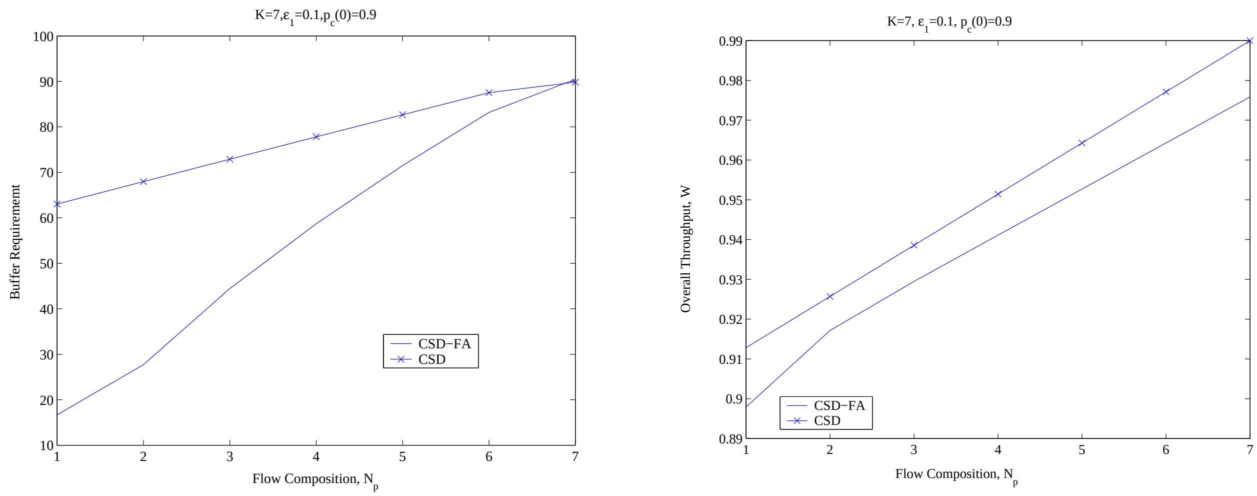
A. Comparison of Throughput and Buffer Requirement of CSD-FA and CSD scheduler

For a given *K* and assuming deterministic one-step channel prediction (i.e., $p_{\hat{0}}$=$p_{\hat{1}}$=1), the metrics *W* and B^{avg} depend on the flow composition, N_p, as well as the channel parameters, ($p_c(0)$,ϵ_1). We illustrate the effects of each parameter on *W* and *B* for *K* = 7, β = 0.01 and ρ = 0.99.

1) Effects of flow composition: We consider the variation of B^{avg} with N_p for $p_c(0)$=0.9 and ϵ_1=0.1 in Fig. 5(a). The corresponding plot for *W* is given in Fig. 5(b).

As the composition of flows with persistent channels (i.e., N_p) is increased, *W* is increased since the accuracy of channel prediction is better for persistent channels. This reduces the likelihood of a wasted slot due to erroneous prediction. Compared to W_{CSD}, the throughput degradation due to flow aggregation is relatively invariant with N_p and is within 2 $\%$.

Since flows with uncorrelated channels have lower delay variation, the average buffer requirement is increased as the proportion of flows with persistent channels is increased. However, $B_{CSD-FA}^{avg} \leq B_{CSD}^{avg}$ due to flow aggregation, and the resultant reduction in buffer requirement is significant (up to 75$\%$) for small values of N_p.



(a) (b)

Fig. 5. Effects of N_p on (a) Average buffer requirement and (b) Overall throughput of various schedulers for $p_c(0)$=0.9 and ϵ_1 = 0.1

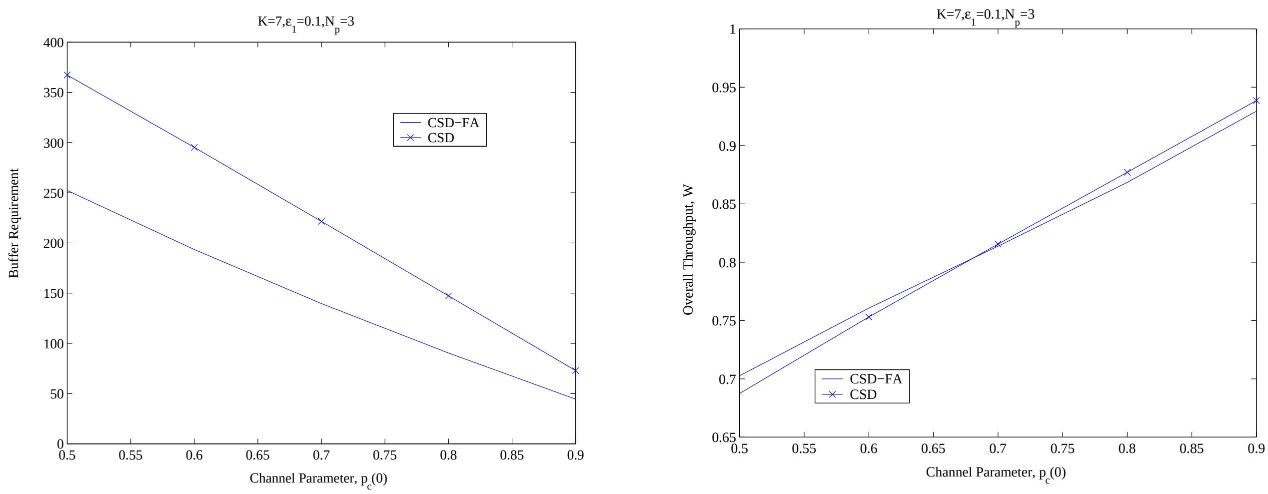
2) Effects of channel quality: Next, we consider the variation of B^{avg} with $p_c(0)$ for N_p=3 and ϵ_1=0.1 in Fig. 6(a). The corresponding plot for *W* is given in Fig. 6(b).

As the channel quality is improved (i.e., $p_c(0)$ is increased), *W* is increased since more transmission attempts will occur and the likelihood of successful transmission is increased. It is interesting to note that flow aggregation actually achieves a slight gain in throughput compared to the CSD scheduler when the channel quality is poor ($p_c(0)$ <0.7). This trend is reversed when channel conditions improve. However, the difference in throughput performance between both schedulers is very marginal (within 2 %).

The buffer requirement is reduced as the channel quality is improved, since delay variation is reduced as flows are more likely to transmit in slots allocated to them. The reduction in buffer requirement as a result of flow aggregation is significant (up to 30%).

3) Effects of channel burstiness: Lastly, we consider the variation of B^{avg} with ϵ_1 for $p_c(0)$=0.9 and N_p=3 in Fig. 7(a). The corresponding plot for *W* is given in Fig. 7(b).

As the channel for Gp_1 flows become less persistent (i.e., ϵ_1 is increased), *W* is reduced since the accuracy of channel prediction is reduced. This increases the likelihood of a wasted slot due to erroneous prediction. Compared to W_{CSD}, the throughput degradation due to flow aggregation is relatively invariant with ϵ_1 and is within 2 %.



(a) (b)

Fig. 6. Effects of $p_c(0)$ on (a) Average buffer requirement and (b) Overall throughput of various schedulers for ϵ_1 = 0.1 and N_p=3

Since the buffer requirement of any flow $\in Gp_2$ is independent of ϵ_1, the metric B^{avg}_{CSD-FA} is determined by the variation of the buffer requirement of flows $\in Gp_1$. It is interesting to note that B^{avg} for both schedulers is reduced initially as ϵ_1 is increased, but is increased with further increase in ϵ_1. However, $B^{avg}_{CSD-FA} \leq B^{avg}_{CSD}$, and the resultant reduction in buffer requirement is significant (up to 45 $\%$).

B. Discussion

A common observation from Section VI-A is a trade-off between throughput and buffer requirements between the CSD-FA and CSD scheduler: The CSD-FA scheduler results in a significant reduction in the wireless receiver buffer requirement at the expense of reduced throughput compared to the CSD scheduler. In fact, since the throughput degradation is marginal compared to the reduction in buffer requirement, the CSD-FA scheduler is effective in maintaining good overall performance.

Our current analysis assumes a simplistic WRR scheduler as the SAP. However, in [23], we study the performance of various loop schedulers in terms of its delay variation and our analysis indicate that the WRR scheduler exhibits the worst-case performance over the entire class of loop schedulers. Hence, the performance of the CSD-FA scheduler can be enhanced by considering

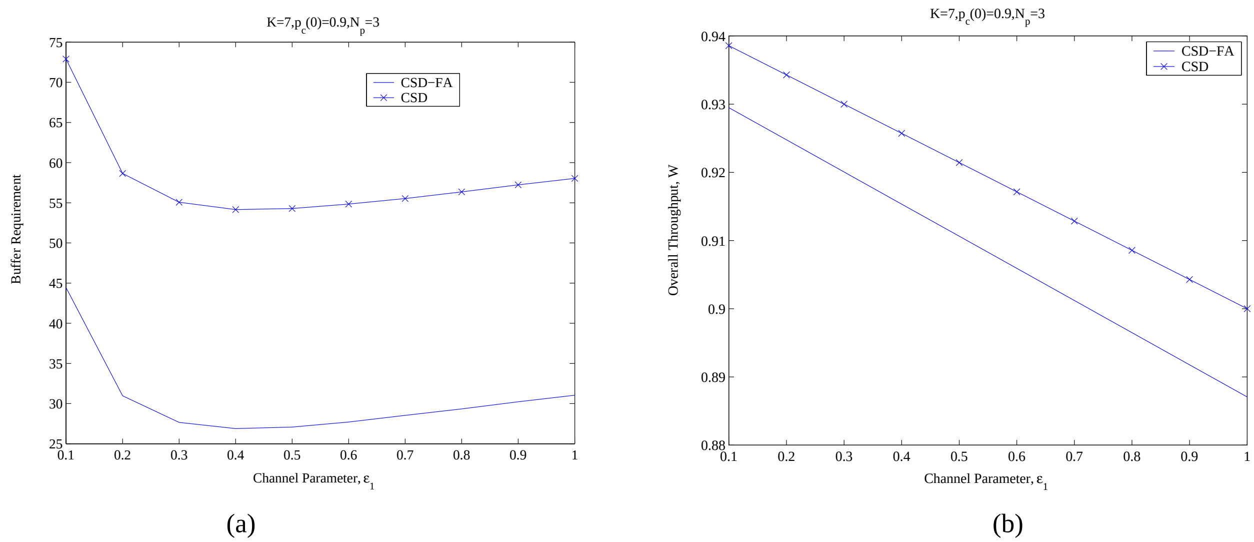


Fig. 7. Effects of ϵ_1 on (a) Average buffer requirement and (b) Overall throughput of various schedulers for $p_c(0)$=0.9 and N_p=3

other loop schedulers for the SAP. Several arbitration schemes are proposed in [19] which may result in performance enhancement over uniform arbitration, which is assumed in our study.

Our analysis in Section V-C assumes that each flow $f \in Gp_2$ is permitted to transmit only in slots allocated to the aggregate flow N_p'. As a result, the overall throughput computed using Eq. (15) is actually a lower bound to the actual achievable throughput, since slots allocated to Gp_1 are actually available to flows $\in Gp_2$. The corresponding buffer requirement computed represents a lower bound since the delay variation of any flow $f \in Gp_2$ is minimized as a result of the assumption.

VII. Conclusions

In this paper, we consider the scheduling problem where data packets from K input flows need to be delivered to K corresponding wireless receivers via a heterogeneous wireless channel. Our objective is to design a wireless scheduler that optimizes the buffer requirement at each wireless receiver while maintaining good throughput performance.

We propose a hybrid scheduler that exploits both the short- and long-term error behavior of the channel of each flow so as to achieve high overall throughput as well as low receiver buffer requirements. The scheduler first partitions the flows according to their long-term error behavior (persistent/uncorrelated) such that flows with uncorrelated channels are fairly aggregated. The aggregated flow is then scheduled alongside the remaining flows with a channel-state dependent scheduler, that exploits the short-term error behavior to maximize channel efficiency.

We compare the overall throughput as well as receiver buffer requirements of our proposed scheduler and a channel-state dependent scheduler. Our proposed scheduler achieves good overall throughput as well as low receiver buffer requirements, thus stressing the importance to exploit the long-term error behavior in addition to the instantaneous channel state in the design of wireless schedulers. These parameters can be evaluated using a measurement-based algorithm proposed in [24].

APPENDIX I
THROUGHPUT PERFORMANCE OF K-FLOW CSD AND (K,N_p) CSD-FA SCHEDULER

We define the following notations:

$$\begin{aligned} p_{d^j} &= \text{Prob(flow } j \text{ will defer a transmission attempt)} \\ p_{s^j} &\equiv \text{Prob(flow } j \text{ will transmit successfully)} \end{aligned}$$

According to the transmission heuristics of a *K*-flow CSD scheduler, a flow will defer any transmission attempt in any slot if it predicts an erroneous channel. In addition, a transmission will be successful only if it predicts an error-free channel and the prediction is correct. Using the above heuristics, we obtain the following expressions for (p_{d^j},p_{s^j}) as follows:

$$\begin{aligned} p_{s^j} &= p_c^j(0)p_{\hat{0}}[1-(1-p_c^j(0))\alpha^j] + (1-p_c^j(0))(1-p_{\hat{1}})p_c^j(0)\alpha^j \\ p_{d^j} &= p_c^j(0)(1-p_{\hat{0}}) + (1-p_c^j(0))p_{\hat{1}} \end{aligned}$$

For the channel defined in Eq. (7), the above parameters can be simplified as follows:

$$\begin{aligned} p_{s^j} &= \begin{cases} p_c(0)p_{\hat{0}}[1-(1-p_c(0))\epsilon_1] + (1-p_c(0))(1-p_{\hat{1}})p_c(0)\epsilon_1, & j \in Gp_1; \\ p_c^2(0)p_{\hat{0}} + (1-p_c(0))(1-p_{\hat{1}})p_c(0), & j \in Gp_2. \end{cases} \\ p_{d^j} &= p_d = p_c(0)(1-p_{\hat{0}}) + (1-p_c(0))p_{\hat{1}} \end{aligned}$$

If $W^{j|a_i}$ denotes the throughput of flow *j* in slot *i*, then for $j \neq a_i$:

$$\begin{aligned}
W^{j|a_i} &= \text{Prob(flow } a_i \text{ will not attempt transmission)} \cdot \text{Prob(flow } j \text{ will transmit successfully)} \cdot \\
&\{ \quad \text{Prob(none of remaining } K\text{-2 flows will attempt transmission)} \\
&+ \quad \frac{1}{2}\text{Prob(1 of remaining } K\text{-2 flows will attempt transmission)} \\
&\cdots \quad \frac{1}{K-1}\text{Prob(all of remaining } K\text{-2 flows will attempt transmission)}\} \\
&= p_d p_{s^j} \{ \binom{K-2}{0} p_d^{K-2} + \frac{1}{2}\binom{K-2}{1} p_d^{K-3}(1-p_d) + \cdots + \frac{1}{K-1}\binom{K-2}{K-2}(1-p_d)^{K-2} \} \\
&= p_d p_{s^j} \sum_{k=1}^{K-2} \frac{\binom{K-2}{k}(1-p_d)^k p_d^{K-2-k}}{k+1} \\
&= \frac{p_{s^j} p_d (1-p_d^{K-1})}{(K-1)(1-p_d)}
\end{aligned}$$

Hence, we have the following:

$$W^{j|a_i} = \begin{cases} p_{s^j}, & j = a_i; \\ \frac{p_{s^j} p_d (1-p_d^{K-1})}{(K-1)(1-p_d)}, & \text{otherwise}; \end{cases} \tag{17}$$

Un-conditioning on a_i, we obtain the following expression for W^j:

$$\begin{aligned}
W^j &= \sum_{a_i=1}^{K} \frac{r^{a_i}}{R} W^{j|a_i} \\
&= \frac{r^j}{R} p_{s^j} + \frac{R-r^j}{R} \frac{p_{s^j} p_d (1-p_d^{K-1})}{(K-1)(1-p_d)}
\end{aligned}$$

If we define the following notation:

$$p_{s_1} = \begin{cases} p_{s^j}, & j \in Gp_1; \\ p_{s^j}, & j \in Gp_2; \end{cases}$$

then we have the following expression for W_{CSD}:

$$\begin{aligned}
W_{CSD} &= \sum_{j=1}^{K} W^j \qquad (18) \\
&= \sum_{j=1}^{N_p} [\frac{r^j}{R} p_{s_1} + \frac{R-r^j}{R} \frac{p_{s_1} p_d (1-p_d^{K-1})}{(K-1)(1-p_d)}] + \sum_{j=N_p+1}^{K} [\frac{r^j}{R} p_{s_2} + \frac{R-r^j}{R} \frac{p_{s_2} p_d (1-p_d^{K-1})}{(K-1)(1-p_d)}]
\end{aligned}$$

A. Rate Homogeneous K-flow CSD Scheduler

In this case, we have r^j = 1 and hence, *R* = *K*. Substituting into Eq. (18), we obtain the following, thus verifying the expression in Eq. (16):

$$\begin{aligned} W_{CSD} &= \sum_{j=1}^{N_p}[\frac{1}{K}p_{s_1} + \frac{K-1}{R}\frac{p_{s_1}p_d(1-p_d^{K-1})}{(K-1)(1-p_d)}] + \sum_{j=N_p+1}^{K}[\frac{1}{K}p_{s_2} + \frac{K-1}{K}\frac{p_{s_2}p_d(1-p_d^{K-1})}{(K-1)(1-p_d)}] \\ &= \frac{1-p_d^K}{(1-p_d)K}[N_p p_{s_1} + (K-N_p)p_{s_2} \end{aligned}$$

B. Rate Homogeneous (K,N_p) CSD-FA Scheduler

According to Section V-B, this is equivalent to a N_p+1 scheduler with the flow and channel parameters as given in Eq. (9). We can use Eq. (17) with *K* = N_p+1 to obtain the conditional throughput, $W^{j|a_i}$, for *j* $\in Gp_1$ as follows:

$$W^{j|a_i} = \begin{cases} p_{s^j}, & j = a_i; \\ \frac{p_{sj}p_d(1-p_d^{N_p})}{N_p(1-p_d)}, & \text{otherwise}; \end{cases}$$

Un-conditioning on a_i, we obtain the following expression for W^j:

$$\begin{aligned} W^j &= \sum_{a_i=1}^{K}\frac{r^{a_i}}{R}W^{j|a_i} \\ &= \frac{1}{K}p_{s_1} + \frac{K-1}{K}\frac{p_{s_1}p_d(1-p_d^{N_p})}{N_p(1-p_d)} \\ &= \frac{p_{s_1}}{K}[\frac{N_p(1-p_d)+(K-1)p_d(1-p_d^{N_p})}{N_p(1-p_d)}] \end{aligned}$$

Since $E[n^j] = \frac{1}{W^j}$, Theorem 1 is verified.

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