Energy-Aware Transmission Control for Wireless Sensor Networks Powered by Ambient Energy Harvesting: A Game-Theoretic Approach

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***Abstract*—We use a Bayesian game-theoretic approach to model transmission control in energy-harvesting wireless sensor networks. In general, the energy state of an energy-harvesting sensor varies more dramatically with time as compared to traditional battery-powered sensors. Therefore, each energy-harvesting sensor is aware of its instantaneous energy state, which is modeled as its private information. Each sensor decides its transmission strategy according to its belief of its opponents' energy states. There exists a Bayesian Nash equilibrium (BNE) where a sensor with energy higher than its energy threshold will decide to transmit at fixed power, and wait otherwise. We show how each sensor determines its threshold to maximize its utility function. Moreover, we show via simulations that the performance of the Bayesian game model is close to that of a perfect-information game where energy states are common information to all sensors. In addition, since the proposed Bayesian game has the advantage of requiring less information exchange overhead, it seems to be more feasible to implement than the perfect-information game.**

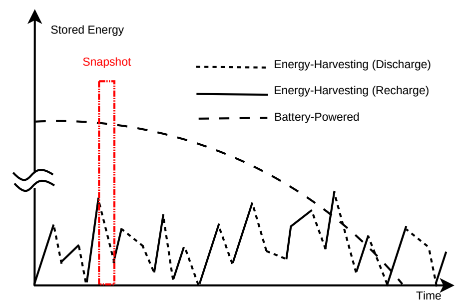


Fig. 1. A comparison of the stored energy in battery-powered WSNs and WSN-HEAP: While it can only deplete with time in battery-powered sensors, the stored energy in energy-harvesting sensors can be replenished by harvesting renewable energy (recharge). Since the energy recharge rate is typically much lower than the rate of consumption, and storage capacity is much smaller, the instantaneous energy state varies more dramatically in energy-harvesting sensors and should be modeled.

I. Introduction

Due to the advancements in the technology of electronic devices, wireless sensor networks (WSNs) have drawn increasing attention recently. There are two types of WSNs according to how the sensors are powered up: (i) battery-powered WSNs, where the battery's energy can only deplete with time; and (ii) WSNs Powered solely by Ambient Energy Harvesting (WSN-HEAP), where the sensor makes uses of renewable energy to replenish its stored energy to maintain its operation [1]. In both types of WSNs, proper energy management is essential to maximize each sensor's utility, which is a function of transmission reliability and operation lifetime. As the instantaneous energy level typically fluctuates more dramatically than that of battery-powered sensors (Fig. 1), it is required in the modeling of energy-harvesting sensors.

In this paper, we use Bayesian game theoretic approach to build an energy-aware transmission control model for WSN-HEAP. There are two advantages to such an approach: firstly, since sensors in WSNs are decentralized devices that make their decisions autonomously, game theory is a good mathematical tool to handle interactions among sensors with selfish behaviors; secondly, we will show that the adoption of a Bayesian game approach can effectively reduce the bandwidth overhead in exchanging information among nodes when compared to a perfect-information game.

II. Related Work

In existing game-theoretic models for battery-powered WSNs, energy management does not necessarily include the instantaneous energy state. Sengupta *e*t al. considered a CDMA-based sensor network where each sensor maximizes the throughput over power [2]. Yuan and Yu proposed a distributed power control and source coding game for multihop WSNs [3], where the objective function of a sensor in the power control game is given by throughput minus power level.

Game-theoretic models have also been developed for energy-harvesting WSNs. Menache and Altman considered a power control game to maximize the sensors' throughput minus cost to replenish the battery [4]. Niyato *e*t al. combined queuing theory and bargaining game to formulate a model for solar-powered WSNs and derived the optimal operating parameters [5]. However, these earlier developed game-theoretic models still lack important considerations in WSN-HEAP. In both [2] and [3], the instantaneous energy state is not taken into consideration. In [4], the energy is replenished only if the battery is depleted, which is not the case in WSN-HEAP. In [5], the interactions among sensors are not analyzed. In this paper, we build non-cooperative game models to analyze WSN-HEAP, and compare the performance between Bayesian (imperfect-information) and perfect-information games for transmission control.

III. System Model

We consider a WSN-HEAP comprising n sensors that communicate directly with a sink. As different sensors may harvest and consume energy at different rates, their stored energy may be different. We denote the stored energy of sensor i by e_i. The transmission from sensor i to the sink is successful only if its Signal-to-Interference Ratio (SIR) γ_i exceeds the SIR threshold γ_i^{th}, namely,

$$\gamma_i(\mathbf{p}) = \frac{g_i p_i}{\sum_{j \neq i} g_j p_j} > \gamma_i^{th}$$

where $\mathbf{p} = (p_1, \ldots, p_n)$, p_i denotes sensor i's transmission power, and g_i denotes the channel gain from sensor i to the sink. The objective (valuation) function v_i of sensor i is the expected duration of successful transmission given by

$$v_i(\mathbf{p}, e_i) = (1 - O_i(\mathbf{p})) \frac{e_i}{P_c(p_i)} \tag{1}$$

where $P_c(\cdot)$ is the power dissipation function for a given p_i, $e_i/P_c(p_i)$ is the discharge time[1] of sensor i, and O_i is the outage probability of sensor i, which is given by

$$O_i(\mathbf{p}) = \mathbf{Prob}(\gamma_i(\mathbf{p}) \leq \gamma_i^{th})$$

We introduce a result on outage probability derived in [6]. Given the channel gain to the sink $g_i = h_i^r * h_i$, where h_i is the Rayleigh fading component and h_i^r is the path loss from sensor i to the sink, the effects of Rayleigh fading will be removed completely when evaluating the average outage probability:

$$O_i(\mathbf{p}) = 1 - \prod_{j \in \mathcal{N} \setminus \{i\}} \frac{1}{1 + \gamma_i^{th} h_j p_j / h_i p_i} \tag{2}$$

Note that the definition is valid even if $p_j = 0$ for some j.

To prevent the sensors from always obtaining positive valuation and transmitting, we assign a cost β_i for each sensor i. The setting of cost will be further discussed in Section VI. We will also refer to (γ_i^{th}, β_i, h_i) as well as $P_c(\cdot)$ as the system parameters of sensor i.

IV. Game-Theoretic Analysis

Since sensors in the WSN-HEAP make decisions autonomously, we can define the following game model:

- A set of sensors $\mathcal{N} = \{1, \ldots, n\}$
- A set of actions (transmission power) for each sensor i: $\mathcal{P}_i \subset [0, p_{max}]$
- A set of types (energy state) for each sensor i: $e_i \in \mathcal{E} = [0, e_{max}]$
- A pure strategy for sensor i is a map $p_i : \mathcal{E} \to \mathcal{P}_i$, which describes an action for each possible type of sensor i.
- A utility function for each sensor i:

$$u_i(\mathbf{p}, e_i) = \begin{cases} (1 - O_i(\mathbf{p})) \frac{e_i}{P_c(p_i)} - \beta_i, & \text{if } p_i > 0 \\ 0, & \text{if } p_i = 0 \end{cases} \tag{3}$$

where $\mathbf{p} = (p_1, \ldots, p_n)$ denotes the strategy profile.

Since each sensor's energy state (type) is private information, the above game is a Bayesian game, where the exact strategy chosen by sensor i remains unknown even though the strategy profile $\mathbf{p}$ is known by all sensors.

A. Bayesian Game and Solution Concept

To solve the Bayesian game, we adopt the Bayesian Nash equilibrium (BNE) solution concept. The sensors have prior beliefs about others' types and maximize their own expected utility given the belief. As [7] suggests, the BNE could provide a good approximation to the globally optimal solution, which we will also show via simulations in Section V.

Definition 1. *(Bayesian Nash Equilibrium) A strategy profile $\mathbf{p}^*(\cdot) = (p_1^*(\cdot), \ldots, p_n^*(\cdot))$ is a BNE if for all $i \in \mathcal{N}$ and for all $e_i \in \mathcal{E}$, we have*

$$p_i^*(e_i) \in \arg\max_{p_i \in \mathcal{P}_i} E_{\mathbf{e}_{-i}}[u_i((p_i, \mathbf{p}_{-i}^*(\mathbf{e}_{-i})), e_i) | e_i]$$

where $E_{e_{-i}}[u_i(\mathbf{p}, e_i)|e_i]$ is the expected utility of sensor i.

Here we denote the strategy profile $\mathbf{p} = (p_i, \mathbf{p}_{-i})$, where $\mathbf{p}_{-i} = (p_1, \ldots, p_{i-1}, p_{i+1}, \ldots, p_n)$ is the strategy profile except for sensor i. Similarly, the vector of energy states except for sensor i is denoted by $\mathbf{e}_{-i}$. The prior belief (i.e., the probability density function) of sensor i about the energy state of sensor j is denoted by $f_{i,e_j}(x)$ for all $j \neq i$. To gain insight into the model, we consider a homogeneous system where the beliefs are independent and identically distributed (i.i.d.). Namely, we have $f_{1,e_j}(x) = \ldots = f_{n,e_j}(x) := f_{e_j}(x)$ and then $f_{e_1}(x) = \ldots = f_{e_n}(x) := f_e(x)$, where e is a random variable representing the energy state of any other sensor with the probability density function $f_e(x)$. We now rewrite the expected utility if $p_i > 0$ as follows:

$$\begin{aligned} & E_{\mathbf{e}_{-i}}[u_i(\mathbf{p}, e_i)|e_i] \\ =& E_{\mathbf{e}_{-i}}[\prod_{j \in \mathcal{N} \setminus \{i\}} \left(1 + \gamma_i^{th} \frac{h_j p_j(e_j)}{h_i p_i(e_i)}\right)^{-1} \frac{e_i}{P_c(p_i(e_i))} - \beta_i | e_i] \\ =& \prod_{j \in \mathcal{N} \setminus \{i\}} \left(E_e[(1 + \gamma_i^{th} \frac{h_j p_j(e)}{h_i p_i(e_i)})^{-1} | e_i]\right) \frac{e_i}{P_c(p_i(e_i))} - \beta_i \end{aligned} \tag{4}$$

We can exchange the product and expectation operators because the beliefs are i.i.d. The expected value with respect to $\mathbf{e}_{-i}$ is now transformed to the one with respect to e. On the other hand, the expected utility is always zero if $p_i = 0$. Therefore, if the expected utility in (4) is greater than zero, then sensor i chooses $p_i > 0$. Otherwise, it chooses $p_i = 0$.

B. BNE Strategy in Threshold Form

In WSNs, adaptive duty cycling is commonly adopted for efficient energy management, where sensors in "good conditions" turn on and transmit, while others turn off. Analogously, in our energy-aware model, sensors with energy state greater than a threshold should transmit, while others wait. In this

[1] In our definition of the discharge time, we have implicitly assumed that the ambient energy harvesting rate is much lower than the rate of energy dissipation in wireless sensors.

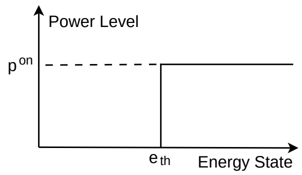


Fig. 2. The threshold-form BNE strategy is a function of transmission power p_i with respect to the energy state e_i.

subsection, we prove that such a threshold-form strategy constitutes a BNE.

Generally speaking, given a energy threshold $e_{th,i}$, sensor i can transmit with any power $p_i > 0$ if its energy state exceeds the threshold (i.e., $e_i > e_{th,i}$). However, we consider the case where sensors either transmit with a fixed power $p^{on} < p_{max}$, or wait with $p_i = 0$ (Fig. 2). We then prove that it is a BNE.

Theorem 1. *If the strategy space $\mathcal{P}_i = \{0, p^{on}\}$ for all i, then there exists a BNE strategy profile $(p_1^*(\cdot), \ldots, p_n^*(\cdot))$ where*

$$p_i^*(e_i) = \begin{cases} p^{on}, & \text{if } e_i > e_{th,i} \\ 0, & \text{otherwise} \end{cases} \tag{5}$$

for all $i \in \mathcal{N}$, where $e_{th,i}$ is a unique energy threshold of sensor i defined as follows:

$$e_{th,i} = \min\{e_i : e_i \in [0, e_{max}], \\ E_{\mathbf{e}_{-i}}[u_i((p^{on}, \mathbf{p}_{-i}^*(\mathbf{e}_{-i})), e_i)|e_i] \geq 0\} \tag{6}$$

In other words, $e_{th,i}$ is the minimum energy state such that the expected utility if $p_i = p^{on}$ remains positive. If $E_{\mathbf{e}_{-i}}[u_i((p^{on}, \mathbf{p}_{-i}^*(\mathbf{e}_{-i})), e_i)|e_i] < 0$ for all $e_i \in [0, e_{max}]$, we let $e_{th,i} = e_{max}$ to stop the sensor from transmitting.

Proof: Using the result derived in (4), we can show that

$$\frac{\partial E_{\mathbf{e}_{-i}}[u_i((p^{on}, \mathbf{p}_{-i}^*(\mathbf{e}_{-i})), e_i)|e_i]}{\partial e_i} \\ = \frac{\prod_{j \in \mathcal{N}\setminus\{i\}} \left(E_e[(1 + \gamma_i^{th} \frac{h_j p_j^*(e)}{h_i p^{on}})^{-1}]\right)}{P_c(p^{on})} > 0 \tag{7}$$

This means that the expected utility if $p_i = p^{on}$ is strictly increasing with e_i. We also have $E_{\mathbf{e}_{-i}}[u_i((p^{on}, \mathbf{p}_{-i}^*(\mathbf{e}_{-i})), 0)] = -\beta_i < 0$. These enable us to obtain the value $e_{th,i}$ defined in (6) such that $E_{\mathbf{e}_{-i}}[u_i((p^{on}, \mathbf{p}_{-i}^*(\mathbf{e}_{-i})), e_i)|e_i] \geq 0$ for all $e_i \geq e_{th,i}$ and $E_{\mathbf{e}_{-i}}[u_i((p^{on}, \mathbf{p}_{-i}^*(\mathbf{e}_{-i})), e_i)|e_i] < 0$ for all $e_i < e_{th,i}$. Note that the threshold $e_{th,i}$ must be unique for each sensor i because the expected utility is strictly increasing. This existence of this threshold ensures that the strategy defined in (5) maximizes the expected utility. ■

The theorem shows that there exists an energy threshold $e_{th,i}$ of each sensor i. Such a threshold-form strategy constitutes a BNE, and the expected utility if $p_i = p^{on}$ can be rewritten as:

$$E_{\mathbf{e}_{-i}}[u_i((p^{on}, \mathbf{p}_{-i}^*(\mathbf{e}_{-i})), e_i)|e_i] \\ = \prod_{j \in \mathcal{N}\setminus\{i\}} \left(\frac{1 + \gamma_i^{th} \frac{h_j}{h_i} F_e(e_{th,j})}{1 + \gamma_i^{th} \frac{h_j}{h_i}}\right) \frac{e_i}{P_c(p^{on})} - \beta_i \tag{8}$$

where $F_e(x)$ is the cumulative density function of the random variable e (i.e., the belief). Notice that this expected utility increases with $e_{th,j}$ for any $j \neq i$, because if $e_{th,j}$ is high, it is less likely for sensor j to transmit and cause interference to sensor i. Hence the expected utility of sensor i is high.

If the minimum in (6) does not exist, $e_{th,i} = e_{max}$; otherwise, the threshold $e_{th,i}$ can be expressed as follows:

$$e_{th,i} = \frac{\beta_i P_c(p^{on})}{\prod_{j \in \mathcal{N}\setminus\{i\}} \left(\frac{1 + \gamma_i^{th} \frac{h_j}{h_i} F_e(e_{th,j})}{1 + \gamma_i^{th} \frac{h_j}{h_i}}\right)} \tag{9}$$

C. The Perfect-Information Game

In this subsection, we consider the more commonly used non-cooperative game model where energy states of the sensors become the common knowledge. We would like to examine to what extent the performance is affected with the use of private information in the Bayesian (imperfect-information) game model as compared to this perfect-information game.

In the earlier proposed Bayesian game, sensors predict the transmission power according to the belief about the others' energy state. However, in perfect-information game, each sensor i knows the exact transmission power $p_j(e_j)$ of any other sensor $j \neq i$, because the energy states are common knowledge. As there is no private information, we shall adopt the commonly used Nash equilibrium (NE) concept.

Definition 2. *(Nash Equilibrium) A strategy profile $\mathbf{p}^* = (p_1^*(e_1), \ldots, p_n^*(e_n))$ is a NE if for all $i \in \mathcal{N}$, we have*

$$p_i^*(e_i) \in \arg\max_{p_i \in \mathcal{P}_i} u_i((p_i, \mathbf{p}_{-i}^*(\mathbf{e}_{-i})), e_i)$$

where u_i is the utility function defined in (3).

Note that in this game, we still consider the same strategy space $\mathcal{P}_i = \{0, p^{on}\}$ of each sensor i as the one in the Bayesian game. Due to the discontinuity of $\mathcal{P}_i$, we are unable to use fixed point theorems to prove the existence of NE. Instead, we adopt the best-response dynamic in [8], and show via simulations that the dynamic always converges to the NE.

The prediction by the perfect-information game model might differ from the one by the Bayesian game model. Typically, the former prediction has information gain over the latter, because the sensors face less uncertainty when the energy states become common knowledge. We will quantify this in the next section.

V. Numerical Results

In this section, we compare the equilibria of four models. The first model is the Bayesian game (Section IV-B). Since the energy states are private information and sensors make decisions in a distributed way, we call it the imperfect-information and distributed (IID) model. The second model is the perfect-information model (Section IV-C). Since the energy states are common knowledge and sensors also make decision in a distributed way, we call it the perfect-information and distributed (PID) model. The third model is a centralized system, where a central controller gathers all the information and provides the optimal solution. However, the computation time

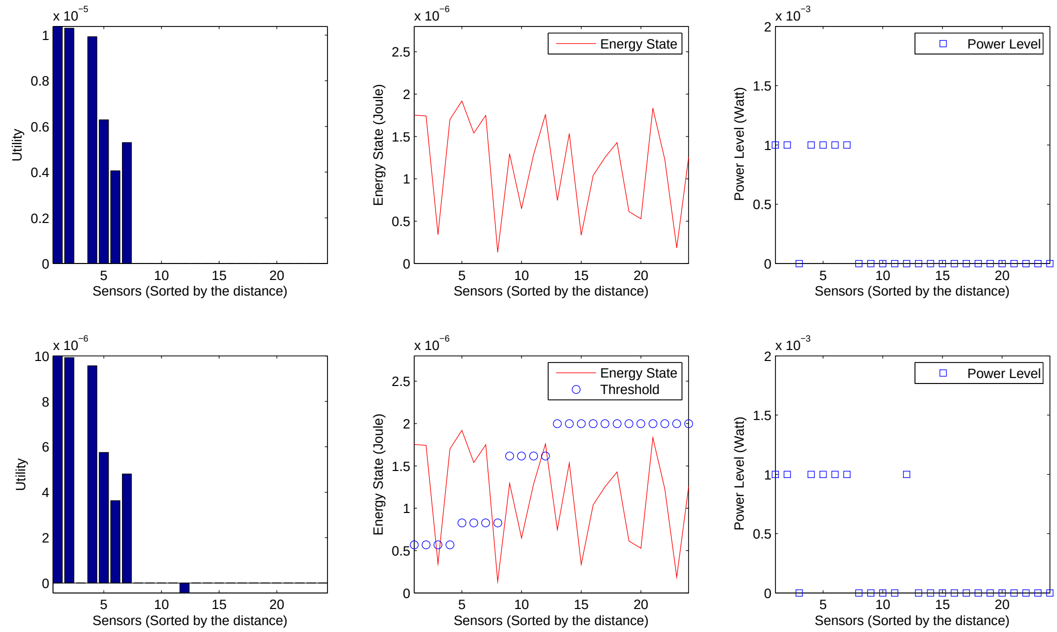


Fig. 3. The comparison between the IID and PID models with identical parameter settings. The nodes are sorted in ascending order by their distance to the sink. The first and second row are results of the PID and IID models respectively. The first column shows the deterministic utility of each sensor in equilibrium. The second column shows the energy state of both models. In the IID model, sensors with the same distance have the same threshold. The third column shows the transmission power of each sensor in the equilibrium. In the PID model, each sensor makes the transmission decision directly, while it is determined by the threshold in the IID model. (Notice that sensor 12 obtains negative utility although its expected utility is positive, which implies that the private information still misleads some sensors and degrades the system performance.)

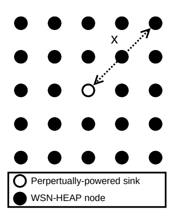


Fig. 4. Illustration of the two-dimensional grid WSN-HEAP deployment comprising 15 sensors and a sink used in our simulations, where x is the maximum distance from the sink to any sensor.

TABLE I
PARAMETER SETTING

	parameter	value
	e_{max}	2e-6 (Joule)
	x	100 (m)
	p^{on}	1 (mW)
	$P_c(p^{on})$	81.8 (mW)
	e	UNI$[0, e_{max}]$
	β_i	5e-6, $\forall i$
	γ_i^{th}	0.1, $\forall i$
	(K, α)	(3.1623e-6, 2)
Fig. 3	n	48
Fig. 5(a)	n	8
Fig. 5(b)	n	8

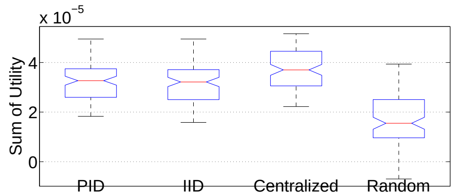
The system parameter values are obtained for the CC2420 radio [9].

grows exponentially with the number of sensors. The fourth model is a random-transmission model where the transmission power of sensors are randomly assigned to either 0 or p^{on}.

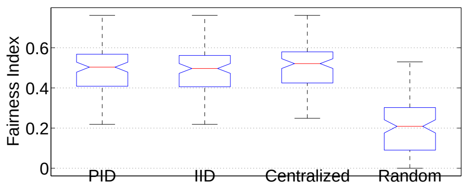
Sensors are deployed in the grid topology (Fig. 4). We consider the free-space path loss model where $h_i = K d_i^{-\alpha}$, K is the propagation factor, d_i is the distance of sensor i to the sink, and α is the path-loss exponent [9]. The other parameter values used in the simulations are shown in Table. I.

Fig. 3 shows the comparison between the equilibria of the IID and PID game models. In the IID model, sensors select the BNE strategy to maximize their expected utility. However, the utility of the IID game model shown in Fig. 3 is not the expected utility in the BNE, but the deterministic utility given that all sensors choose the equilibrium strategy. The deterministic utility represents the prediction outcome by the model. On the other hand, in the PID model, sensors can directly calculate their deterministic utility based on the observation of other sensors' transmission power. We then compare the prediction outcomes by both models.

Firstly, we find that the prediction by the IID model is similar to the one by the PID model, but not exactly the same. Namely, most of sensors are not misled by the private information. Only some sensors choose different actions between the PID and IID models. Secondly, in the IID model, the decision of transmission power depends mainly on the distance to the sink. However, the energy state still plays an important role. For example, sensor 4 has the same distance as sensor 3, but it transmits because of its high energy state. Thirdly, those sensors with $e_{th,i} = e_{max}$ have no chance to transmit even if their energy approaches the maximum. But we can enable



(a) Sum of Utility



(b) Fairness

Fig. 5. The comparison between the PID model, the IID model, the centralized model, and the random-transmission model in view of the sum of utility and Jain's fairness index. Notice that the sum of utility also represents the overall throughput of the system.

them to transmit by lowering the cost. We will discuss this further in Section VI.

In Fig. 5(a) and Fig. 5(b), we examine the efficiency of the IID model in the form of a box plot. The data is obtained from 100 independent experiments with randomly selected energy levels, while other parameters are fixed as in Table. I. We adopt two performance metrics to evaluate the system performance: 1) sum of utility, and 2) Jain's fairness index [10] given by

$$\text{Jain's Fairness Index} = \frac{(\sum_{i\in\mathcal{N}} u_i)^2}{n\sum_{i\in\mathcal{N}} u_i^2}$$

In Fig. 5(a) and Fig. 5(b), both IID and PID models have similar performance because the notches overlap and are significantly better than the random-transmission model. Although information gain exists, it is not significant.

VI. Discussions

A. Feasibility and Implementation Issues

Although game theory provides an excellent analytic tool, the excessive overhead due to information exchange still makes implementation difficult. However, in this paper, the adoption of Bayesian game avoids such a problem. Consider a system where sensors play the game many times. If it is the perfect-information game, the information about energy states needs to be updated every time. However, if it is the Bayesian game, the BNE strategy profile need not be updated unless the system parameters or the beliefs change. Therefore, sensors can still behave as in the perfect-information model, but need not update the information frequently.

B. Choice of System Parameters

In the simulations, we choose $\gamma_i^{th} = 0.1$ for all i because we need to enable more than one sensor to access the sink to show the difference. Such a choice can be justified by considering a CDMA-based WSN where sensors transmit via spread spectrum. However, this is not necessarily the case, and the models can be adapted to other kinds of WSNs by choosing the appropriate parameters.

We implicitly assume that the distribution of energy state is known by the sensors. However, in reality, the belief of sensors may not match the real distribution. In this case, the information gain may become larger due to the wrong belief.

Finally, the cost β_i is determined by some external factors but not controlled by sensor i itself. For example, the cost can be set lower if the transmission has high priority, if the sensor has not been transmitting for a long time, or if the energy-harvesting rate is so low that the energy states are generally low. If so, then the threshold becomes lower and the sensor has more chance to transmit. The way to set the cost depends on the application of WSNs. In the simulations, we set the same cost because we consider a homogeneous WSN.

VII. Conclusion

We present a Bayesian game-theoretic model for transmission control in WSN-HEAP. Because the energy states are private information, sensors determine the transmission power according to their prior belief of others' energy states. We prove that the Bayesian Nash equilibrium exists and the BNE strategy of each sensor can be expressed in the threshold form: if the energy state exceeds the energy threshold, then the sensor transmits with a fixed power; otherwise, the sensor waits. The Bayesian game model has similar performance to the perfect-information game model, but the overhead is significantly reduced, making it more feasible for implementation.

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References

[1] W. K. G. Seah, Z. A. Eu, and H. P. Tan. Wireless sensor networks powered by ambient energy harvesting (wsn-heap)-survey and challenges. In *Wireless VITAE*, pages 1–5, 2009.

[2] S. Sengupta, M. Chatterjee, and K. A. Kwiat. A game theoretic framework for power control in wireless sensor networks. *IEEE Transactions on Computers*, 59(2):231 –242, 2010.

[3] J. Yuan and Yu W. Distributed cross-layer optimization of wireless sensor networks: A game theoretic approach. In *IEEE GLOBECOM*, pages 1 –5, 2006.

[4] I. Menache and E. Altman. Battery-state dependent power control as a dynamic game. In *WiOPT*, pages 242–250, 2008.

[5] D. Niyato, E. Hossain, and A. Fallahi. Sleep and wakeup strategies in solar-powered wireless sensor/mesh networks: Performance analysis and optimization. *IEEE Transactions on Mobile Computing*, pages 221–236, 2007.

[6] S. Kandukuri and S. Boyd. Optimal power control in interference-limited fading wireless channels with outage-probability specifications. *IEEE Transactions on Wireless Communications*, 1(1):46–55, 2002.

[7] H. J. Lee, H. Kwon, A. Motskin, and L. Guibas. Interference-aware mac protocol for wireless networks by a game-theoretic approach. In *IEEE INFOCOM*, pages 1854 –1862, 19-25 2009.

[8] D. Fudenberg and D. K. Levine. *The theory of learning in games*. The MIT press, 1998.

[9] H. P. Tan, A. Valera, and W. Koh. Transmission power control in 2-d wireless sensor networks powered by ambient energy havesting. In *IEEE PIMRC*, 2010.

[10] R. Jain, D. Chiu, and W. Hawe. A quantitative measure of fairness and discrimination for resource allocation in shared computer systems. *DEC Research Report TR-301*, 1984.