Profit-Maximizing Incentive for Participatory Sensing

Tie Luo*, Hwee-Pink Tan* and Lirong Xia†
*Institute for Infocomm Research (I^{2}R), Singapore
†Rensselaer Polytechnic Institute (RPI), USA
Email: {luot, hptan}@i2r.a-star.edu.sg, xial@cs.rpi.edu

***Abstract*—We design an incentive mechanism based on all-pay auctions for participatory sensing. The organizer (principal) aims to attract a high amount of contribution from participating users (agents) while at the same time lowering his payout, which we formulate as a profit-maximization problem. We use a contribution-dependent prize function in an environment that is specifically tailored to participatory sensing, namely incomplete information (with information asymmetry), risk-averse agents, and stochastic population. We derive the optimal prize function that induces the maximum profit for the principal, while satisfying strict individual rationality (i.e., strictly have incentive to participate at equilibrium) for both risk-neutral and weakly risk-averse agents. The thus induced profit is demonstrated to be higher than the maximum profit induced by constant (yet optimized) prize. We also show that our results are readily extensible to cases of risk-neutral agents and deterministic populations.**

***Index Terms*—Mechanism design, Bayesian game, all-pay auction, perturbation analysis, network economics, crowdsensing.**

I. Introduction

The global proliferation of smartphones has spurred the recent emergence of participatory sensing paradigm, in which the public crowd rather than professionals undertake various sensing activities with inbuilt sensors on their smartphones. This paradigm relieves the need for deploying and maintaining costly wireless sensor networks, yet can achieve pervasive spatiotemporal coverage, making it appealing enough to have spawned a large number of projects such as PEIR [1], ParkNet [2] and Ear-phone [3], to name a few.

The crowdsourcing nature and lacking of intrinsic motivation in many participatory sensing applications, however, render *incentive* the foremost challenge to the viability of participatory sensing. Although a variety of incentives in similar contexts have been discussed, including micro-payment [4], reputation [5], and fun [6], we deem auction [7] to be an excellent candidate for designing incentive mechanisms, as it solves the challenging issue of *pricing* participants' effort, endogenously (rather than by an external authority) by leveraging the market power under a game setting. Indeed, we are not alone in holding this opinion; some prior studies [8]–[10] were carried out in this regard. The common central theme of these studies is to determine a sound wage for participants using reverse auctions, with certain objectives such as retaining auction losers [8], minimizing cost and ensuring truthful bidding [9], [10].

In this paper, we provide a mechanism that incentivizes participatory sensing in the spirit of *all-pay auctions*. In a standard all-pay auction, the auctioneer announces an indivisible good for sale and the bidders tender their respective bids; the highest bid wins the good but *all* the bidders will have to pay their bids. This seemingly weird form is in contrast to first-price or Vickery (second-price) auctions which belong to the more intuitive category of *winner-pay* auctions. However, the all-pay notion precisely reflects a participatory sensing campaign that is conducted in the following form: rather than remunerating *every* participant who contributes to the sensing campaign, as in [8]–[10], the campaign organizer allocates a single prize for all participants to compete for; the one who makes the highest contribution will win the prize but all the participants will have exerted (paid) their *irrevocable* effort (bids). Compared to other auctions that require trusting winners to actually and exactly pay their bids (i.e., exert their "promised" effort) *after* winning the auction, all-pay auctions in the above form is less stringent and thus may be more suitable for participatory sensing applications which are often conducted in a largely ad-hoc manner.

All-pay auctions are not limited to modeling participatory sensing but can be applied to crowdsourcing and contests [11]–[14] as well. There is also a wealth of literature in auction theory [7], [15]. Our work differs from prior work in a number of important aspects enumerated below, which as well constitute our main contributions.

Profit maximization: Generally speaking, profit involves both revenue and cost. In standard auctions (including first-price, second-price, and all-pay versions) [7], [15] and many of their variants (including contests and crowdsourcing) [11]–[14], the auctioned item or the prize is fixed ex-ante and hence the cost to the auctioneer is constant. This is also the case in [9] when describing its platform-centric model; although its user-centric model has a variable cost component (as an immediate result of paying different sets of users) in the organizer's utility, the *maximum* organizer utility cannot be achieved due to NP-hardness. The work [10] minimizes the total payment to participants (i.e., cost), but does not factor revenue into the objective function; instead, a fixed revenue (in the form of the quality of service) was taken as a constraint to satisfy. Moreover, the all-pay auction approach that we take also sets our work apart from [8]–[10].

In this paper, we allow the organizer (henceforth referred to as the principal[1]) to set the prize as a *function* of the

[1] We adopt the terminology of "principal" and "agent" due to the analogy of participatory sensing to the principal-agent problem which concerns the difficulties in motivating the agents to act in the best interest of the principal while the two parties have different interests and asymmetric information.

winner's contribution in order to induce the maximum *profit* which is defined as the total contribution acquired from all the users (henceforth referred to as agents) less the contribution-dependent prize. We demonstrate that such a contribution-dependent prize can induce higher profit than *optimal* constant-prize mechanisms.

Risk-averse agents: One most common (and often implicit) assumption in the related literature is that agents are risk neutral. Loosely speaking, it means that agents are indifferent between a sure-win $50 reward and a $100 reward that is conditioned on flipping a coin. This is not always true in real scenarios, particularly when agents are risk averse, preferring the guaranteed $50 reward in the above example. Therefore, we pose the problem by facing the principal with risk-averse agents who have stronger reluctance to lose than willingness to win the same amount. Not only does this set up a more realistic problem, but it also lends generality to our solution which subsumes risk-neutral agents as well.

On the other hand, this brings about the challenge that the celebrated revenue equivalence theorem[2] [16], [17], which is a powerful facility to analyzing many auctions, breaks in our model. Another challenge is that there is generally no explicit solution to risk aversion models. To overcome these difficulties, we use *perturbation analysis* introduced by [18].

Stochastic population: Another common and implicit assumption in standard auctions and the majority of related work, is that the number of agents is common knowledge ex ante. This does not apply to many participatory sensing scenarios where agents barely know how many other agents are actively participating; even the principal, who may be able to retrieve (e.g., from database) the number of agents who have registered for the sensing campaign, is still unable to know the number of actual participating agents—indeed, the discrepancy between the two numbers can be substantial.

In view of this realistic constraint, we assume a stochastic setting where the number of agents is uncertain. It is noteworthy that allowing for uncertainty bestows us an additional advantage: a participatory sensing campaign thus modeled can accommodate *ad hoc entry*. That is, agents can enter the auction anytime, instead of being required to place bids simultaneously as in other auctions. This has practical implication as the public crowds, who are participatory sensing users, behave by and large in an ad-hoc manner.

Incomplete information with information asymmetry: As participatory sensing involves a potentially large population of agents who typically are strangers to one another, we consider an incomplete information setting with information asymmetry: agents are at the *interim stage* where each of them knows exactly his own *type*[3] but only probabilistically about other agents' types, while the principal is at the *ex ante stage* where he does not know each agent's exact type but only has the same probabilistic knowledge as the agents have. This Bayesian game setting is in contrast to the complete-information game setting which is used by [9] (particularly the platform-centric model) and [19] for example.

Altogether, our profit-maximization objective along with the main assumptions enumerated above, which are specifically tailored to the context of participatory sensing, constitute a unique (and presumably meaningful) problem to tackle, and underpin the main contributions of this work.

To the best of our knowledge, this paper also represents the first work that explicitly introduces all-pay auctions into participatory sensing for incentive mechanism design.

Finally, besides maximizing profit for the principal, our mechanism satisfies *strict individual rationality* for both risk-neutral and weakly risk-averse agents. That is, such agents expect strictly positive payoff at equilibria; or in other words, they strictly have incentive to participate.

The rest of the paper is organized as follows. Section II reviews the literature and Section III describes our model. In Section IV, we solve for the optimal prize function that induces the maximum profit, and prove the strict individual rationality. Section V provides a case study in order to derive an intuitive understanding of our analysis and to demonstrate key properties of our results. Section VI concludes.

II. Related Work

Arguably the first study that addresses incentive for participatory sensing using (reverse) auctions is [8], in which mobile users sell their sensing data to the organizer by bidding their desired selling prices. Those who bid lower than a (hidden) threshold—set by the organizer—will win and can sell at their respective bid prices, while the rest will lose and not get paid. A main feature thereof was to "subsidize" losers with virtual credits to reduce their in-effect bid prices but not affect their actual pay if they win. This helps retain users and keep the winning price competitive (i.e., sufficiently low).

Two other incentive models were investigated by [9]. One is called a platform-centric model, in which a central platform allocates a constant reward to be divided among all the users in proportion to their respectively planned sensing times. The constant reward is optimized in terms of maximizing the platform utility, by formulating a Stackelberg game under a complete-information setting where users' unit costs are common knowledge. The other model is called a user-centric model, in which each user claims a task set to undertake together with his desired payment, and the platform selects a subset of users (as winners) to perform the tasks and pays them no lower than their respective bids. This is essentially a reverse auction. However, maximizing the platform utility turns out to be NP-hard, and as such the design objective was to ensure each user to report his true cost for payment.

Likewise, [10] also lets users report their unit costs, but the service provider will decide on each users' participation level (e.g., data sampling rate) as well as his payment. A reverse auction mechanism is proposed to minimize the total payment while satisfying a given quality of service. It also achieves *incentive compatibility* as in [9], i.e., users declare their types (unit costs) truthfully.

Our model allows each agent to (strategically) decide on his own participation level, which we deem to be more natural. Our all-pay auction mechanism satisfies strict individual rationality,

[2] The theorem states that any auction will lead to the same expected revenue if the auction satisfies a set of standard assumptions.

[3] As a standard term in Bayesian games and mechanism design, "type" refers to a player's private information or signal, such as his valuation of the auctioned item, his skill level, marginal cost, etc.

while incentive compatibility is technically unrelated because in our case the agent type (cost, skill, etc.) is endogenized into his bidding strategy (participation level).

A crowdsourcing website offering diverse online tasks for users to undertake and earn reward was studied by [13] as a *matching market*, where users select tasks based on their skill sets and the offered (constant) prizes. The problem is tackled using the revenue equivalence theorem by virtue of adopting standard assumptions such as risk neutrality. In addition, the number of users is assumed to be known. These assumptions are also adopted by [9], [10].

Along a different spirit, [20] proposes a market-based incentive scheme using a demand-and-supply model: participants are not only data suppliers but also service consumers who demand information service provisioned from processing the contributed data. Therefore, instead of offering monetary incentive, the service provider allocates consumable service to participants in accordance with their data contribution levels, in such a way that ensures fairness and maximizes social welfare.

In the vast economic literature, all-pay auctions with complete information was analyzed by Baye et al. [19], where every bidder's valuation of the item on sale is common knowledge. In contrast, [12], [14] study all-pay auctions with incomplete information, but focus on investigating whether a single prize or multiple prizes (with the same lump sum) should be allocated in order to maximize the quality of the highest k submissions [12] or the aggregate quality [14]. Along a similar line, [11] shows that in a standard all-pay auction, the highest bid is at least half the sum of all the bids.

With a known number of risk-neutral players, [21], [22] examine a paradoxical behavior where a reduction in the reward or an increase in cost may actually increase the expected sum of bids or the highest bid. On the other hand, yet still with a deterministic population, [23] studies risk-averse players with a fixed item on sale in first-price auctions.

For first and second-price auctions, [24] assumes that the the number of bidders is known to the auctioneer and studies the difference in revenue between concealing and revealing this number to bidders. That study does not characterize equilibrium bidding strategies (whereas this study does). A similar problem was studied by [25] but using a different approach called a maxmin expected utility model. Equilibrium bidding strategies for standard winner-pay (instead of all-pay) auctions are characterized by [26] and [7] using different methods, e.g., the revenue equivalence theorem [7] which is not applicable to our model. Recently, [27] studied all-pay auctions with a random number of identical bidders whose types are equal, whereas we consider agents of heterogeneous types which reflect the diversity of participants more realistically in the context of participatory sensing.

III. Model

We consider a general class of participatory sensing applications in which a principal announces some sensing activity and calls for participation to a potentially large population of agents. As participating in the activity (e.g., sending sensory information from smartphones) will incur cost (e.g., battery drain, network charge, time and effort commitment) to agents, the principal announces a monetary prize as the incentive, to be rewarded to the agent who makes the highest contribution by the end of the activity, or by the end of a prescribed period if the activity is conducted periodically. Agents have the flexibility to start contributing anytime (i.e., ad hoc entry) after the call for participation is announced by the principal.

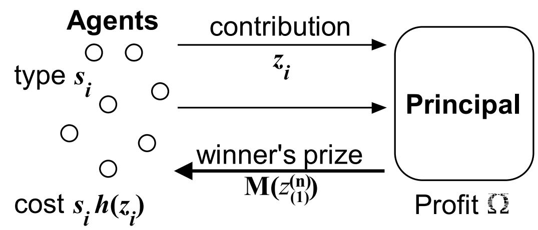


Figure 1: An all-pay auction based model for participatory sensing.

An agent i is characterized by a unique type $s_i \in [\underline{s}, \bar{s}]$ which is private information; the type could be the marginal cost of participation, as used by [9], [10], or a (monotone decreasing) function of agent's *skill*, as used by [13]. There is a continuum of agent types independently drawn from $[\underline{s}, \bar{s}]$ according to an atomless, right-continuous cumulative distribution function $F(s)$, which is common knowledge.[4] Each agent decides on the amount of contribution z_i to make, which is his strategy. The amount z_i could be a simple measure of quantity alone (e.g. sensing time [9]) or a compound measure of both quality and quantity (e.g., cumulative information quality [28]).[5] Making the contribution z_i will incur a cost $s_i h(z_i)$ to agent i. Unlike the conventional linear cost model used in the vast literature, we allow for more generality by inserting a modulator function $h(\cdot)$ to model the (possibly nonlinear) impact of an agent's contribution strategy on his cumulative cost—for instance, an analogy could be drawn to the Matthew effect (the rich get richer and the poor get poorer). We assume that $h(\cdot)$ is continuous, $h(0) = 0$ and $h' > 0$. Key notations are summarized in Fig. 1.

The principal is pre-committed to reward a monetary prize to the agent who makes the highest contribution. The prize is not fixed but is a continuous function $M(\cdot)$ of the maximum of $\tilde{n}$ agents' contributions, $z_{(1)}^{(\tilde{n})}$, following the notation in order statistics. The rationale is to examine if we can induce a higher *profit* which is defined as the total contribution acquired from all the agents less the contribution-dependent prize (i.e., revenue less cost), or formally,

$$\Omega(\tilde{n}, \boldsymbol{z}) = \sum_{i=1}^{\tilde{n}} z_i - M(z_{(1)}^{(\tilde{n})}). \tag{1}$$

where $\tilde{n}$ is the (uncertain) number of agents.

Agents are risk averse, characterized by a von Neumann-Morgenstern (vNM) utility function $u(\cdot)$ that is twice differentiable and satisfies $u(0) = 0, u' > 0, u'' < 0$ (i.e., more

[4] In participatory sensing, such an independent-private-value model [7] with Bayesian belief can be realized using historical information or, in the absence, by adopting the uniform distribution, as similarly discussed in [10].

[5] In practice, z_i can be measured by agents' smartphones or by the principal, depending on the application. In the latter case, the value is fed back to the corresponding agent (e.g., on his phone) continually to keep him informed and for him to decide when to stop contributing.

reluctant to lose than willing to gain), and accordingly an agent i's payoff function is formulated as

$$\pi_i(s_i, \boldsymbol{z}) = \begin{cases} u(M(z_i) - s_i h(z_i)), & \text{if } z_i > z_j, \forall j \neq i; \\ u(-s_i h(z_i)), & \text{otherwise.} \end{cases} \quad (2)$$

Note that our model subsumes risk-neutral agents by simply letting $u(x) = x$. We ignore the situation of multiple winners, which happens with zero probability since F is atomless and z is a monotone function of s [7]. The number of agents $\tilde{n}$ follows a probability mass function $p_n := \Pr(\tilde{n} = n), n = 0, 1, 2...$, which is common knowledge.

Problem statement: Our objective is to design an incentive mechanism based on the above all-pay auction such that (a) the expected profit of the principal at equilibrium is maximized, i.e., $\max_{M(\cdot)} \Omega^*$ where $\Omega^* := \mathbb{E}_{\tilde{n}}[\Omega(\tilde{n}, \boldsymbol{z}^*)]$ and "*" signifies equilibrium, and (b) *strict individual rationality* is satisfied, i.e., $\pi_i^* := \pi_i(s_i, \boldsymbol{z}^*) > 0$ iff $z_i^*(s_i) > 0$, for both risk-neutral and weakly risk-averse agents.[6] In other words, the expected payoff of each such agent is strictly positive if he contributes. Compared to the canonical definition of individual rationality which only requires non-negativity ($\pi_i^* \geq 0$), our mechanism implies a stronger motivation to participants.

IV. Analysis

We first analyze the equilibrium contribution strategy z^* for a stochastic population of (weakly or strongly) risk-averse agents, and then solve for the optimal prize function $M(\cdot)$ that maximizes the principal's expected profit Ω^* facing weakly risk-averse agents. The results are then extended to risk-neutral agents and deterministic populations.

A. Equilibrium contribution strategy

Definition 1 (Bayesian Nash Equilibrium)**.** *A (pure strategy) Bayesian Nash equilibrium of our all-pay auction model is a strategy profile $\boldsymbol{z}^* := (z_1^*, z_2^*, ...)$ that satisfies*

$$\pi_i(z_i^*(s_i), z_{-i}^*(s_{-i})) \geq \pi_i(z_i, z_{-i}^*(s_{-i})), \forall z_i, \forall i.$$

In words, at a Bayesian Nash equilibrium, each agent plays a (pure) strategy that maximizes his expected payoff given his belief about other agents' types and that other agents play their respective equilibrium strategies.

Lemma 1. *There exists a pure strategy Bayesian Nash equilibrium for the all-pay auction defined in Section III.*

Proof: Our auction model corresponds to a Bayesian game setting where a higher type induces a lower bid. Hence whenever every agent other than an arbitrary agent i uses a decreasing strategy, i.e., each $z_j \in z_{-i}$ is a monotone decreasing function of type s_j, agent i's best response z_i is also decreasing in s_i [14]. This satisfies the Spence-Mirrlees single-crossing property [29] and thus a pure strategy Nash equilibrium (PSNE) exists in every finite-action Bayesian game. Further, given that the functions $M(\cdot), h(\cdot), u(\cdot)$ are all continuous and there is a continuum of agent type s, the game induced from our all-pay auction model has continuous payoffs and a continuum of strategies. Therefore, according to [29], there exists a sequence of PSNE in finite-action games that converges to a PSNE of the continuum-action game. ■

In the sequel, we simply say "equilibrium" to refer to pure strategy Bayesian Nash equilibrium. Also, we focus on symmetric equilibrium in which all agents adopt the same strategy function at equilibrium.[7]

Analyzing the equilibrium strategy involves expressing an agent's expected payoff using the probability that he wins the auction. Related to this probability and in a stochastic population setting, *contingent bidding*[8] was proposed by [26] but is not applicable to participatory sensing, because the principal does not know the number of contributing agents before they actually contribute, let alone that agents cannot make contingent effort as effort is irrevocable. In our case—a Bayesian auction in which a higher type induces a lower bid, an agent's strategy z_i is a monotone decreasing function of s_i, and determines the probability that he wins the auction when there are $\tilde{n} = n$ agents to be $\Pr(z_i > z_j, \forall j \neq i | \tilde{n} = n) = (1 - F(s_i))^{n-1}$. Therefore, under a stochastic population, the winning probability for an arbitrary agent of type s is

$$P(s) = \sum_n p_n (1 - F(s))^{n-1}. \quad (3)$$

However, one should not be alluded to reckoning that, once agent i's equilibrium strategy when fixing $\tilde{n} = n$ can be obtained, say as $z_i^*(n)$, his strategy in the stochastic case is immediately given by $z_i^* = \sum_n p_n z_i^*(n)$. The reason is that, simplistically speaking, z_i^* is not an affine optimizer (in the sense of best-response strategy) of π_i^*, as analogous to Jensen's inequality. Indeed, even in a standard auction where bidders are risk-neutral, the equilibrium bidding strategy with uncertain population is not simply a p_n-weighted sum, as shown by [7]. Note that, however, the proof technique used by [7] does not apply here because it uses the revenue equivalence theorem which assumes risk neutrality.

Now we write an agent's expected payoff, using (2), as

$$\begin{aligned} \pi(s, z) &= u(M(z) - h(z)s)P(s) + u(-h(z)s)(1 - P(s)) \\ &= P(s)\big[u(\alpha) - u(-\beta)\big] + u(-\beta) \end{aligned} \quad (4)$$

where we denote $\alpha := M(z) - h(z)s$ and $\beta := h(z)s$.

Lemma 2. *In an all-pay auction with incomplete information and a stochastic population of risk-averse agents, given a contribution-dependent prize function $M(z)$, the equilibrium strategy $z^*(s)$ is determined by*

$$\begin{aligned} &\int_s^{\bar{s}} \Big[P(s_1)\Big(u'(\alpha^*) - u'(-\beta^*)\Big) + u'(-\beta^*)\Big] h(z^*(s_1))\, \mathrm{d}s_1 \\ &= P(s)\big[u(\alpha^*) - u(-\beta^*)\big] + u(-\beta^*) \end{aligned} \quad (5)$$

where $\alpha^* = M(z^*) - h(z^*)s$ and $\beta^* = h(z^*)s$.

Proof: Because each agent is playing his best response at equilibrium, his payoff π^* is also the solution to the optimization problem $\max_z \pi(s, \boldsymbol{z}^*)$. Furthermore, we know

[6] In this paper, individual rationality (IR) refers to interim IR.

[7] In Bayesian games, a strategy is a *function* mapping from type space to action/strategy space.

[8] In contingent bidding, each bidder submits a *list* of bids like "I bid z_1 if there are n_1 bidders, z_2 if n_2, ..." After collecting all the lists, the auctioneer knows the actual number of bidders and will use the contingent bids corresponding to that number (n_k).

$\pi^* = \pi(s, \boldsymbol{z}^*(s))$ if the equilibrium strategy $z^*(s)$ is given. Therefore, we use the envelope theorem [30] to obtain

$$\frac{\partial \pi^*}{\partial s} = -P(s)\big[u'(\alpha^*)h(z^*) - u'(-\beta^*)h(z^*)\big] - u'(-\beta^*)h(z^*).$$

Since the highest-type agent never wins the auction at equilibrium, he will contribute zero and reap zero surplus, i.e., $z^*(\bar{s}) = 0$ and $\pi^*(\bar{s}) = 0$. Therefore, integrating both sides from s to $\bar{s}$ leads to the l.h.s. being $\int_s^{\bar{s}} \frac{\partial \pi^*}{\partial s} = -\pi^*(s)$. The result is then proven by plugging in (4). ■

In general, there is no explicit expression of equilibrium strategy when agents are risk-averse or the prize is not constant; otherwise, Lemma 2 yields a closed-form solution, demonstrated later by Corollary 3.[9]

B. Profit maximization

When every agent adopts equilibrium strategy $z^*(\cdot)$, it follows from (1) that

$$\Omega(\tilde{n}, \boldsymbol{z}^*) = \tilde{n} \int_{\underline{s}}^{\bar{s}} z^* \, \mathrm{d}F - M(z^{*(\tilde{n})}_{(1)})$$

where $\tilde{n}$ is temporarily treated as given. Because $z^{*(\tilde{n})}_{(1)} = z^*(s^{(\tilde{n})}_{(\tilde{n})})$ where $s^{(\tilde{n})}_{(\tilde{n})}$ is the $\tilde{n}$-th order of all agent types (i.e., the minimum of all the $\tilde{n}$ types), and from order statistics we know that the c.d.f. of $s^{(\tilde{n})}_{(\tilde{n})}$ is $1 - (1 - F(s))^{\tilde{n}}$, hence

$$M\big(z^{*(\tilde{n})}_{(1)}\big) = \int_{\underline{s}}^{\bar{s}} M(z^*(s)) \, \mathrm{d}[1 - (1 - F(s))^{\tilde{n}}].$$

Therefore, the expected profit at equilibrium is given by

$$\begin{aligned} \Omega^* &= \mathbb{E}_{\tilde{n}}[\Omega(\tilde{n}, \boldsymbol{z}^*)] \\ &= \sum_n np_n \int_{\underline{s}}^{\bar{s}} [z^* - M(z^*)(1-F)^{n-1}] \, \mathrm{d}F. \end{aligned} \tag{6}$$

To maximize this profit, we need to derive an explicit expression of $M(z^*)$ from (5), which unfortunately cannot be obtained as aforementioned. To deal with this challenge, we consider the case of *weak risk aversion*[10] and employ *perturbation analysis* introduced by [18] to solve the problem.

For weakly risk-averse agents, the vNM utility function can be written as

$$u(x) = x + \epsilon u_1(x) + O(\epsilon^2), \qquad 0 < \epsilon \ll 1.$$

where ϵ is the risk aversion parameter and $0 < \epsilon \ll 1$ means weak risk aversion. The function u_1 satisfies $u_1(0) = 0, u_1'' < 0$, and $u_1' > -\frac{1}{\epsilon}$, so that $u' > 0$ and $u'' < 0$.

Then, using perturbation analysis, we can rewrite

$$\begin{aligned} u(\alpha) &= \alpha + \epsilon u_1(\alpha) + O(\epsilon^2) \\ &= \alpha + \epsilon u_1(\alpha_{rn}) + O(\epsilon^2), \\ u(-\beta) &= -\beta + \epsilon u_1(-\beta) + O(\epsilon^2) \\ &= -\beta + \epsilon u_1(-\beta_{rn}) + O(\epsilon^2), \\ u'(\alpha) &= 1 + \epsilon u_1'(\alpha_{rn}) + O(\epsilon^2), \\ u'(-\beta) &= 1 + \epsilon u_1'(-\beta_{rn}) + O(\epsilon^2), \end{aligned} \tag{7}$$

where the subscript rn signifies the risk-neutral case, i.e.,

$$\alpha_{rn} = M_{rn}(z_{rn}) - h(z_{rn})s, \quad \beta_{rn} = h(z_{rn})s.$$

Notation: We have mentioned that we use superscript star (*) to indicate equilibrium (induced by any given prize function). In the sequel, we use overhead circle (○) to indicate the *optimal* equilibrium (induced by the optimal, i.e., profit-maximizing, prize function).

Theorem 1. *In an all-pay auction with incomplete information and a stochastic population of weakly risk-averse agents, the optimal prize function that induces the maximum profit for the principal is given by*

$$\begin{aligned} \mathring{M}(z) = \frac{1}{P(\mathring{s}(z))} \Big[& \mathring{s}(z)h(z) - \mathring{A}(\mathring{s}(z)) + \\ & \int_0^z \mathring{B}(\mathring{s}(z_1))h(z_1) \, \mathrm{d}\mathring{s}(z_1) \Big]. \end{aligned} \tag{8}$$

In the above, $\mathring{s}(z)$ *is the inverse function of* $\mathring{z}(s)$ *which is the optimal equilibrium strategy, induced by* (8) *as*

$$\mathring{z}(s) = g^{-1}\Big(\frac{aF'(s)}{G'(s)s + G(s)\mathring{B}(s)}\Big), \tag{9}$$

where $g(\cdot) := h'(\cdot)$, $h'' > 0$, $a = \sum_n np_n$,

$$\begin{aligned} G(s) &= \int_{\underline{s}}^{s} F'(s_1) \frac{\sum_n np_n(1 - F(s_1))^{n-1}}{\sum_n p_n(1 - F(s_1))^{n-1}} \, \mathrm{d}s_1, \\ \mathring{A}(s) &= \epsilon P(s)[u_1(\mathring{\alpha}_{rn}) - u_1(-\mathring{\beta}_{rn})] + \epsilon u_1(-\mathring{\beta}_{rn}), \\ \mathring{B}(s) &= \epsilon P(s)[u_1'(\mathring{\alpha}_{rn}) - u_1'(-\mathring{\beta}_{rn})] + 1 + \epsilon u_1'(-\mathring{\beta}_{rn}). \end{aligned} \tag{10}$$

Proof: Rewrite (5) using perturbation expressions (7), as

$$\begin{aligned} &\int_s^{\bar{s}} \Big[\epsilon P(s_1)\Big(u_1'(\alpha^*_{rn}) - u_1'(-\beta^*_{rn})\Big) + 1 + \epsilon u_1'(-\beta^*_{rn})\Big] \\ &\qquad \times h(z^*) \, \mathrm{d}s_1 \\ &= P(s)M(z^*) + \epsilon P(s)[u_1(\alpha^*_{rn}) - u_1(-\beta^*_{rn})] - \beta^* \\ &\qquad + \epsilon u_1(-\beta^*_{rn}), \text{ or} \end{aligned}$$

$$\int_s^{\bar{s}} B^*(s_1)h(z^*) \, \mathrm{d}s_1 = P(s)M(z^*) + A^*(s) - \beta^* \tag{11}$$

where

$$\begin{aligned} A^*(s) &= \epsilon P(s)[u_1(\alpha^*_{rn}) - u_1(-\beta^*_{rn})] + \epsilon u_1(-\beta^*_{rn}), \\ B^*(s) &= \epsilon P(s)[u_1'(\alpha^*_{rn}) - u_1'(-\beta^*_{rn})] + 1 + \epsilon u_1'(-\beta^*_{rn}) \end{aligned} \tag{12}$$

are functions of s only, given z^*_{rn} and $M_{rn}(\cdot)$. Hence,

$$M(z^*) = \frac{1}{P(s)}\Big(\beta^* - A^*(s) + \int_s^{\bar{s}} B^*(s_1)h(z^*) \, \mathrm{d}s_1\Big). \tag{13}$$

Substituting (13) into (6) yields

$$\begin{aligned} \Omega^* = \int_{\underline{s}}^{\bar{s}} \Big[& z^* \sum_n np_n - \frac{\sum_n np_n(1-F)^{n-1}}{\sum_n p_n(1-F)^{n-1}} \times \\ & \Big(\beta^* - A^*(s) + \int_s^{\bar{s}} B^*(s_1)h(z^*) \, \mathrm{d}s_1\Big)\Big] \, \mathrm{d}F \\ = \int_{\underline{s}}^{\bar{s}} & \Big[az^*F' - G'(s)(\beta^* - A^*(s))\Big] \, \mathrm{d}s - \\ & \int_{\underline{s}}^{\bar{s}} \Big[G'(s) \int_s^{\bar{s}} B^*(s_1)h(z^*) \, \mathrm{d}s_1\Big] \, \mathrm{d}s, \end{aligned}$$

[9] Closed-form solutions are also obtained in Theorem 1 and Corollary 2; note, however, that they pertain to the *optimal* equilibrium and not all equilibria.

[10] Weak risk aversion is in fact a common phenomenon in practice. As an intuitive example, although one would prefer a guaranteed \$49 over half-likely \$100, he would not prefer a guaranteed \$20 over half-likely \$100.

where we note that z^* is no longer a function of n as in the usual deterministic population setting. Integrating the second term by parts gives

$$\Big(G(s)\int_s^{\bar{s}} B^*(s_1)h(z^*)\,\mathrm{d}s_1\Big)\Big|_{\underline{s}}^{\bar{s}} - \int_{\underline{s}}^{\bar{s}} G(s)(-B^*(s)h(z^*))\,\mathrm{d}s$$
$$= \int_{\underline{s}}^{\bar{s}} G(s)B^*(s)h(z^*)\,\mathrm{d}s.$$

Thus, the principal's objective can be rewritten as

$$\max_{z^*} \Omega^* = \int_{\underline{s}}^{\bar{s}} \Big[az^*F' - G'(s)(\beta^* - A^*(s)) - G(s)B^*(s)h(z^*)\Big]\,\mathrm{d}s. \quad (14)$$

Here we have changed from optimizing over $M(z)$ to over z^*, because the principal essentially aims to induce an optimal equilibrium *strategy* (by the means of an optimal prize function) that maximizes his profit. Solving (14) is equivalent to maximizing the integrand which we denote by γ. Using the first-order condition (FOC) with respect to z^* and fixing s, we have

$$\frac{\partial \gamma}{\partial z^*} = aF' - sG'(s)h'(z^*) - G(s)B^*(s)h'(z^*) = 0, \quad (15)$$

from which the optimum solution (9) follows (with z^* changed to $\mathring{z}$ and B^* to $\mathring{B}$). Then, (8) is obtained by substituting $\mathring{s}(z)$ for s in (13) (with A^* changed to $\mathring{A}$) and converting the limits of integral.

In addition, for the FOC to lead to a valid maximizer, we require $\frac{\partial^2\gamma}{\partial z^{*2}} < 0$ which implies $[G'(s)s+G(s)B^*(s)]h''(z^*) > 0$. Under weak risk aversion, $\epsilon \ll 1$, so $B^*(s) > 0$. Furthermore, because $G'(s) > 0$ and $G(s) > 0$, it requires $h'' > 0$. ■

Since $\mathring{A}(s)$ and $\mathring{B}(s)$ as defined in (10) assume $\mathring{M}_{rn}(\cdot)$ and $\mathring{z}_{rn}$, which are the optimal prize function and the induced optimal equilibrium strategy in the corresponding risk-neutral case, we solve for these two in Corollary 2.

Recalling our problem statement in Section III which involves both profit maximization and strict individual rationality, we state the following result.

Theorem 2. *The equilibrium strategy (as given by Lemma 2) satisfies strict individual rationality for both risk-neutral and weakly risk-averse agents.*

Proof: From the proof of Lemma 2 we know that $\pi^*(s)$ equals the l.h.s. of (5), which is rewritten in the proof of Theorem 1 as the l.h.s. of (11). Hence,

$$\pi^*(s) = \int_s^{\bar{s}} B^*(s_1)h(z^*(s_1))\,\mathrm{d}s_1.$$

Rearrange $B^*(s)$, as defined in (12), as

$$B^*(s) = \epsilon[P(s)u_1'(\alpha_{rn}^*) + (1-P(s))u_1'(-\beta_{rn}^*)] + 1.$$

For risk-neutral agents, $B^*(s) = 1$ because $\epsilon = 0$. For weakly risk-averse agents, $u_1' > -\frac{1}{\epsilon}$ and hence

$$B^*(s) > \epsilon[-\frac{P(s)}{\epsilon} - \frac{1-P(s)}{\epsilon}] + 1 = 0.$$

Since $h(z) > 0$ iff $z > 0$, therefore for both agents it holds that $\pi^* > 0$ iff $z^* > 0$, i.e., an agent who makes nonzero contribution[11] expects strictly positive payoff at equilibrium. ■

C. Risk-neutral agents

The results for risk-neutral agents can be derived conveniently from the general results presented in the preceding section.

Corollary 1. *In an all-pay auction with incomplete information and a stochastic population of* **risk-neutral** *agents, given a contribution-dependent prize function $M_{rn}(z)$, the equilibrium strategy $z_{rn}^*(s)$ is determined by*

$$\int_s^{\bar{s}} h(z_{rn}^*(t))\,\mathrm{d}t = M_{rn}(z_{rn}^*)P(s) - h(z_{rn}^*)s. \quad (16)$$

Proof: Immediately follows from Lemma 2 by substituting $u(x) = x$ and $u' = 1$ into (5). ■

Corollary 2. *In an all-pay auction with incomplete information and a stochastic population of* **risk-neutral** *agents, the optimal prize function that induces the maximum profit for the principal is given by*

$$\mathring{M}_{rn}(z) = \frac{1}{P(\mathring{s}_{rn}(z))}\Big[\mathring{s}_{rn}(z)h(z) + \int_0^z h(z_1)\,\mathrm{d}\mathring{s}_{rn}(z_1)\Big] \quad (17)$$

where $\mathring{s}_{rn}(z)$ is the inverse function of $\mathring{z}_{rn}(s)$ which is the optimal equilibrium strategy, induced by (17) *as*

$$\mathring{z}_{rn}(s) = g^{-1}\Big(\frac{aF'(s)}{G'(s)s + G(s)}\Big) \quad (18)$$

where $g(\cdot) := h'(\cdot)$, $h'' > 0$.

Proof: Letting $\epsilon = 0$ leads to $\mathring{A}(s) = 0$ and $\mathring{B}(s) = 1$. Substituting these into Theorem 1 yields the result. ■

D. Deterministic population

When the number of agents is known ex ante, we present the results with respect to risk-averse and risk-neutral agents separately, as one particular set of the results will be used later.

Risk averse, deterministic population (RA-DP): In this case, we have expressions simplified to $a = n, G'(s) = nF'(s), G(s) = nF(s)$, $P(s) = (1-F(s))^{n-1}$. Substituting these into Lemma 2 and Theorem 1, we obtain the optimal equilibrium strategy and optimal prize function for the RA-DP case. We omit full details for brevity.

Risk neutral, deterministic population (RN-DP): Substituting the same set of simplified expressions as above, and additionally $u(x) = x$ and $\epsilon = 0$, into Corollary 1 and Corollary 2, yields the result for RN-DP (this result is spelt out below as it will be referred to by our case study in Section V): the optimal prize function is given by

$$\mathring{M}_{rn,dp}(z_{rn,dp}) = \frac{\mathring{s}(z_{rn,dp})h(z_{rn,dp}) + \int_0^{z_{rn,dp}} h(z_1)\,\mathrm{d}\mathring{s}(z_1)}{(1-F(\mathring{s}(z_{rn,dp})))^{n-1}}, \quad (19)$$

[11]The highest-type agent will not participate for otherwise he will expect negative utility if he contributes any $z > 0$. In fact, this agent arises with zero probability because $F(s)$ is atomless.

which induces the optimal equilibrium strategy

$$\mathring{z}_{rn,dp}(s) = g^{-1}\Big(\frac{F'}{sF'+F}\Big). \tag{20}$$

Finally and in particular, if the prize is constant, we have an explicit and compact result on the equilibrium strategy:

Corollary 3. *In an all-pay auction with incomplete information and a* **deterministic population** n *of* **risk-neutral** *agents, given a* **constant prize** M_0*, the equilibrium strategy is given by*

$$z^*_{rn,dp,cp} = h^{-1}\Big((n-1)M_0\int_s^{\bar{s}}(1-F)^{n-2}\frac{F'(t)}{t}\,\mathrm{d}t\Big) \tag{21}$$

Proof: Substituting M_0 into (16) for $M_{rn}(\cdot)$ and differentiating (16) with respect to s yields

$$-h(z^*_{rn,dp,cp}) = M_0P'(s) - h(z^*_{rn,dp,cp}) - s\frac{\mathrm{d}h(z^*_{rn,dp,cp})}{\mathrm{d}s}$$

$$\Rightarrow \qquad z^*_{rn,dp,cp} = h^{-1}\Big(-M_0\int_s^{\bar{s}}\frac{P'(t)}{t}\,\mathrm{d}t\Big),$$

plugging $P(s) = (1-F)^{n-1}$ into which leads to (21). ■

In this (simplest) case, (1) simplifies to

$$\Omega^*_{rn,dp,cp} = n\int_{\underline{s}}^{\bar{s}} z^*_{rn,dp,cp}\,\mathrm{d}F - M_0, \tag{22}$$

and hence the principal can maximize his profit by optimizing over M_0 using the first-order condition, given the distribution function $F(\cdot)$ and modulator function $h(\cdot)$. This is demonstrated in Section V.

Remark: Interestingly, it might be counter-intuitive to note that agents' contribution strategy $\mathring{z}_{rn,dp}$ given by (20) is independent of population size n, whereas the strategy under a constant prize, as in (21) and all the standard auctions, depends on n. The latter is quite easily understood by intuition: a larger number of agents imply a more competitive auction and hence an agent should adjust his strategy accordingly. So, why agents become indifferent to this number now?

The reason, which is equally interesting, is as follows. In standard all-pay auctions, increasing population size n causes a *dampening effect* where agents shade their bids downward to minimize loss because the auction becomes harder to win. However, now that the principal has the privilege to functionize the prize, he could do better by *endogenizing* the number n into the prize function (see (19)) in such a way that each agent would maintain his bid (contribution).

Of course, the prize should not be raised insofar as to incur deficit; indeed, the principal reaps a larger profit than using a (optimal) constant prize, which we show in Section V.

V. Case Study

We consider a participatory sensing campaign in which a government office (the principal) wants to acquire certain information such as real-time noise or traffic data from smartphone users (agents) citywide continuously. Each agent is characterized by his type—his marginal cost of contributing the data—derived from (as the inverse of) his skill or competency. We assume that agents' competency levels are independently and uniformly drawn from a continuous interval $(0,1]$, and hence the agent type s is independently distributed as per c.d.f. $F(s) = 1-\frac{1}{s}$, $s\in[1,\infty)$. An agent i who makes a cumulative contribution of z_i will incur a cost of $s_ih(z_i)$, where the modulator function $h(z) = z^w, w>1$, describes a superlinear increase of cumulative cost when contribution accrues. This corresponds to, for instance, the following scenarios: (1) $h(z)$ is the time or effort needed to produce a certain value of information (VoI) z (e.g., in terms of entropy) that exhibits diminishing gain from time/effort, and hence to produce more VoI consumes increasingly more time/effort; (2) the contribution z relates linearly to an agent's time/effort but making contribution interferes more and more with the agent's regular life and work activities as z increases, or he becomes increasingly more impatient as time/effort is continually spent on the sensing activity. In the sequel, we take the quadratic function (i.e, $w=2$) for numeric calculation.

Agents are weakly risk averse, characterized by a vNM function $u(x) = x-\epsilon x^2$. The number of contributing agents is uncertain, but is known to follow a uniform probability mass function $p_n = \frac{1}{N}$, $n = 2,3,...,N+1$. The maximum, $N+1$, could be the total number of registered smartphone users, retrieved from a registrant database.

In the following, we compare the maximum profit induced by our mechanism (contribution-dependent prize, or CDP) with that induced by the optimal constant-prize mechanism (OCP), under two settings, RA-SP and RN-DP (spelt out in the headings below). There are obviously two other possible combinations, but we choose to demonstrate using these two "extremes" in the sense of complexity.

A. Risk averse, stochastic population (RA-SP)

To apply Theorem 1, first we calculate $G'(s) = \frac{1}{s^2}[\sum_{n=2}^{N+1}n(\frac{1}{s})^{n-1}]/[\sum_{n=2}^{N+1}(\frac{1}{s})^{n-1}]$. By denoting $t := \frac{1}{s}\in(0,1]$ and noting that $\sum_n nt^{n-1} = (\sum_n t^n)'_t$,

$$G'(s(t)) = t^2\frac{\big[\frac{t^2(1-t^N)}{1-t}\big]'_t}{\frac{t(1-t^N)}{1-t}} = t^2\cdot\frac{\frac{t^2-2t+(N+2)t^{N+1}-(N+1)t^{N+2}}{(1-t)^2}}{\frac{t(1-t^N)}{1-t}}$$

$$= t^2\big[\tfrac{(N+1)t_1^N}{1-t_1^N} - \tfrac{1}{1-t_1^N} - \tfrac{1}{1-t_1}\big],$$

$$\therefore G(s(t)) = \textstyle\int_1^s G'(s_1)\,\mathrm{d}s_1 = \int_t^1 G'(s(t_1))\frac{1}{t_1^2}\,\mathrm{d}t_1$$

$$= \textstyle\int_t^1\big[\frac{(N+1)t_1^N}{1-t_1^N} - \frac{1}{1-t_1^N} - \frac{1}{1-t_1}\big]\,\mathrm{d}t_1.$$

Since $a = \frac{N+3}{2}$, it follows from Corollary 2 that

$$\mathring{z}_{rn}(s) = (N+3)/[4G'(s)s^3 + 4G(s)s^2],$$
$$\mathring{M}_{rn}(s) = [h(\mathring{z}_{rn})s - \mathring{\pi}_{rn}(s)]/P(s)$$

where we denote $\mathring{\pi}_{rn}(s) := -\int_0^{\mathring{z}_{rn}} h(z_1)\,\mathrm{d}\mathring{s}_{rn}(z_1)$. Because $u_1(x) = -x^2$, we can spell out

$$\mathring{A}(s) = \epsilon P(s)[2\mathring{M}_{rn}(s)h(\mathring{z}_{rn})s - \mathring{M}^2_{rn}(s)] - \epsilon h^2(\mathring{z}_{rn})s^2$$
$$= -\epsilon[\tfrac{1}{P}\mathring{\pi}^2_{rn} + 2(\tfrac{1}{P}-1)\mathring{\pi}_{rn}h(\mathring{z}_{rn})s + (\tfrac{1}{P}-1)h^2(\mathring{z}_{rn})s^2],$$
$$\mathring{B}(s) = -2\epsilon P(s)\mathring{M}_{rn}(s) + 1 + 2h(\mathring{z}_{rn})s$$
$$= (2-2\epsilon)h(\mathring{z}_{rn})s + 2\epsilon\mathring{\pi}_{rn} + 1.$$

Now we can apply Theorem 1 to obtain the optimal equilibrium strategy as

$$\mathring{z}(s) = \frac{(N+3)/4}{G'(s)s^3 + G(s)s^2[(2-2\epsilon)h(\mathring{z}_{rn})s + 2\epsilon\mathring{\pi}_{rn} + 1]}$$

and the maximum profit (following from (14) as a shortcut)

$$\mathring{\Omega} = \int_1^{\infty} \left[\tfrac{(N+3)\mathring{z}}{2s^2} - G'(s)\big(h(\mathring{z})s - \mathring{A}(s)\big) - G(s)\mathring{B}(s)h(\mathring{z})\right] \mathrm{d}s$$

where $P = \frac{s+1}{2s^2}$ and $h(x) = x^2$.

We compute the above expressions using numerical methods and symbolic computation tools with *Mathematica*.[12]

Constant Prize: When the prize is a constant M_0, we solve for equilibrium strategy z^* by substituting M_0 into (11) for $M(z^*)$ and differentiating (11) with respect to s, yielding

$$\frac{\mathrm{d}h(z^*_{cp})}{\mathrm{d}s} + (1 - B^*(s))h(z^*_{cp}) = M_0 P'(s) + A^{*\prime}(s).$$

Note that $A^*(s)$ and $B^*(s)$ in the above assume $z^*_{rn,cp}$, which is given by

$$z^*_{rn,cp} = -\frac{M_0}{2}\int_s^{\infty} \frac{P'(s_1)}{s_1}\,\mathrm{d}s_1 = (\frac{1}{8s^2} + \frac{1}{6s^3})M_0,$$

obtained by following the proof of Corollary 3.

To maximize the profit, we first solve for z^*_{cp} by symbolically solving the above differential equation together with $h(x) = x^2$. Then, substituting z^*_{cp} into $\Omega^*_{cp} = \sum n p_n \int_{\underline{s}}^{\bar{s}} z^*_{cp}\,\mathrm{d}F - M_0$, we numerically compute the maximum.[13]

Results: Fig. 2 compares the profit induced by CDP (our mechanism) to that by OCP. In Fig. 2a, we fix risk aversion parameter $\epsilon = 0.1$ and vary $N = 2, 3, ..., 6$ (recall that $n = 2, 3, ..., N{+}1$). The graph indicates that CDP constantly outperforms OCP for all N. Due to the toolkit limit on computation, we choose to demonstrate a larger range of n in the deterministic case (next subsection) where we discuss results in greater detail.

In Fig. 2b, we fix $N = 5$ and vary ϵ from 0.05 to 0.3 with step size 0.05.[14] We find that risk aversion affects profit positively in both mechanisms: both CDP and OCP gain slightly higher profit (by about 8-15%) than with risk-neutral agents. This is because risk-averse agents bid more aggressively for fear of losing the auction. Furthermore, we find that risk aversion has a bigger positive impact on the profit of CDP than of OCP, and the profit of CDP exhibits convexity as ϵ gets larger. This is manifested by the two tangent lines drawn at $\epsilon = 0.05$ and $\epsilon = 0.3$; on the other hand, OCP exhibits an approximately linear relation to risk aversion.

B. Risk neutral, deterministic population (RN-DP)

The equilibrium strategy in this case follows from (20) to be $\mathring{z} = \frac{1}{2s^2}$, and accordingly $\mathring{s} = \frac{1}{\sqrt{2z}}$. The optimal prize function then follows from (19) to be $\mathring{M}(z) = (2z)^{2-\frac{n}{2}}/6$ or equivalently, in terms of s, $\mathring{M}(s) = s^{n-4}/6$.

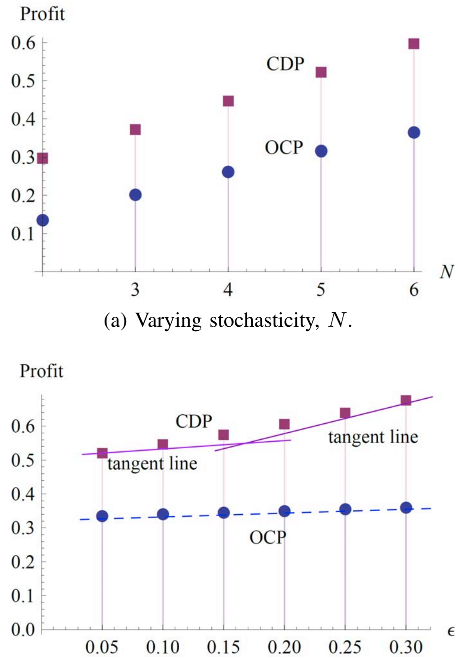
The resultant maximum profit is obtained from (6) (with slight adaptation to DP) as

$$\mathring{\Omega} = n\int_1^{\infty} [\mathring{z}(s) - \mathring{M}(z)(1-F)^{n-1}]\,\mathrm{d}F = \frac{1}{8}n.$$

[12]Relevant tools include *Integrate*, *NIntegrate*, and *InverseFunction*.

[13]Relevant tools include *DSolve*, *Integrate* and *Maximize*.

[14]The value of ϵ should not be too large for otherwise it will violate the assumption of weak risk aversion. In the computation we have ensured $u_1'(x) > -\frac{1}{\epsilon}$.



(a) Varying stochasticity, N.

(b) Varying risk aversion parameter, ϵ.

Figure 2: Profit comparison in a RA-SP setting. CDP: contribution-dependent prize (our mechanism); OCP: optimal constant prize.

Constant Prize: On the other hand, if the prize is a constant M_0, Corollary 3 gives the equilibrium strategy $z^* = s^{-\frac{n}{2}}\sqrt{\frac{n-1}{n}M_0}$. The profit at equilibrium can thus be derived from (22) as

$$\Omega^* = n\int_1^{\infty} z^*\,\mathrm{d}F - M_0 = \frac{2\sqrt{n(n-1)M_0}}{n+2} - M_0,$$

and the maximum profit can be then obtained using FOC, as

$$\Omega^*_{max} = \frac{n(n-1)}{(n+2)^2}$$

which is achieved when $M_0 = \frac{n(n-1)}{(n+2)^2}$ (i.e., revenue is the double of cost).[15]

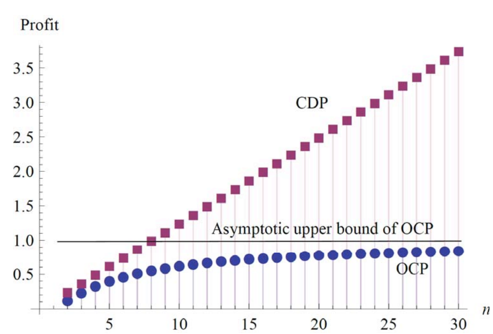


Figure 3: Profit comparison under a RN-DP setting, by varying the deterministic number of agents, n.

Results: We compare these two mechanisms in Fig. 3. The closed-form expressions permit demonstrating the result over

[15]In the case of RN-DP with CP, the revenue equivalence theorem applies, and hence can also be used to solve for the equilibrium strategy and *revenue*. However, when using the theorem, note that the revenue collected from each agent is $h(z)$ and not z, in order to obtain the same result.

a larger range of n which we vary up to 30. Besides that CDP outperforms OCP as observed in Fig. 2a, we additionally see that the profit of CDP exhibits a linearly increasing trend whereas the profit of OCP exhibits diminishing gain as n increases. Even more promising is the asymptotic behavior: as n goes to infinity, Ω^*_{max} approaches 1 whereas $\mathring{\Omega}$ maintains the linear growth with n, which is highly desirable. This is partially explained by the remark in Section IV-D (and we provide further explanation in Section VI) where CDP endogenizes n into the prize function to negate the dampening effect from which, however, OCP suffers.[16]

VI. Concluding Remarks

Intuitively, why would a contribution-dependent prize perform better than an optimized constant prize? Philosophically speaking, the former offers one more degree of freedom for the principal to maneuver, and thus he should naturally be able to do better than without. Technically speaking, we have explained that the principal uses the prize function to endogenize the number of agents to offset the dampening effect of a larger population size on agents' contribution. Another reason is that the prize function allows the principal to leverage risk-averse agents' fear of losing auctions to incentivize agents to contribute, as demonstrated in Section V-A.

Put in a realistic setting, CDP also has an advantage of counteracting *skewed* belief, where (some or all) agents view other agents to be of higher-type (if the type is cost-alike) or lower-type (if the type is skill-alike) than their actual types, or in intuitive terms, agents may underestimate others to be "weaker" than they actually are. Therefore, if the prize is constant, an agent has incentive to *reserve effort* (i.e., make less contribution) in order to reduce the *winning margin* since he can only win the fixed prize regardless of how much he outdoes the runner-up. In a worse case, the total contribution (revenue) may not even cover the principal's fixed cost and thus result in deficit. In contrast, CDP suppresses this disincentive of exerting effort by offering extra reward if the winner overdoes; on the other hand, when contribution turns out to be low, the cost (prize) automatically shrinks and thereby avoid deficit.

To summarize, this paper provides an incentive mechanism based on all-pay auctions in participatory-sensing or crowd-sensing contexts. This mechanism accommodates incomplete information with information asymmetry, risk-averse agents and stochastic population. As such, it can apply generally or be extended to other human-centric networking environments.

References

[1] M. Mun, S. Reddy, K. Shilton, N. Yau, J. Burke, D. Estrin, M. Hansen, E. Howard, R. West, and P. Boda, "PEIR, the personal environmental impact report, as a platform for participatory sensing systems research," in *ACM MobiSys*, 2009, pp. 55–68.

[2] S. Mathur, T. Jin, N. Kasturirangan, J. Chandrasekaran, W. Xue, M. Gruteser, and W. Trappe, "ParkNet: drive-by sensing of road-side parking statistics," in *ACM MobiSys*, 2010, pp. 123–136.

[3] R. Rana, C. T. Chou, S. Kanhere, N. Bulusu, and W. Hu, "Ear-phone: An end-to-end participatory urban noise mapping system," in *ACM/IEEE IPSN*, 2010.

[4] S. Reddy, D. Estrin, M. Hansen, and M. Srivastava, "Examining micro-payments for participatory sensing data collections," in *UbiComp*, 2010.

[5] Y. Zhang and M. van der Schaar, "Reputation-based incentive protocols in crowdsourcing applications," in *IEEE INFOCOM*, 2012.

[6] K. Han, E. A. Graham, D. Vassallo, and D. Estrin, "Enhancing motivation in a mobile participatory sensing project through gaming," in *IEEE SocialCom*, 2011.

[7] V. Krishna, *Auction theory*, 2nd ed. Academic Press, 2009.

[8] J.-S. Lee and B. Hoh, "Sell your experiences: A market mechanism based incentive for participatory sensing," in *IEEE PerCom*, 2010.

[9] D. Yang, G. Xue, X. Fang, and J. Tang, "Crowdsourcing to smartphones: Incentive mechanism design for mobile phone sensing," in *ACM MobiCom*, 2012.

[10] I. Koutsopoulos, "Optimal incentive-driven design of participatory sensing systems," in *IEEE INFOCOM*, 2013.

[11] S. Chawla, J. D. Hartline, and B. Sivan, "Optimal crowdsourcing contests," in *ACM-SIAM Symposium on Discrete Algorithms*, 2012.

[12] N. Archak and A. Sundararajan, "Optimal design of crowdsourcing contests," in *30th International Conference on Information Systems*, 2009.

[13] D. DiPalantino and M. Vojnovic, "Crowdsourcing and all-pay auctions," in *10th ACM conference on Electronic commerce*, 2009, pp. 119–128.

[14] B. Moldovanu and A. Sela, "The optimal allocation of prizes in contests," *American Economic Review*, vol. 91, no. 3, pp. 542–558, 2001.

[15] P. Klemperer, *Auctions: Theory and Practice*. Princeton University Press, 2004.

[16] R. Myerson, "Optimal auction design," *Mathematics of Operations Research*, vol. 6, no. 1, pp. 58–73, 1981.

[17] J. Riley and W. Samuelson, "Optimal auctions," *American Economic Review*, vol. 71, no. 3, pp. 381–392, 1981.

[18] G. Fibich and A. Gavious, "Asymmetric first-price auctions - a perturbation approach," *Mathematics of Operations Research*, vol. 28, no. 4, pp. 836–852, 2003.

[19] M. R. Baye, D. Kovenock, and C. G. de Vries, "The all-pay auction with complete information," *Economic Theory*, vol. 8, no. 2, pp. 291–305, 1996.

[20] T. Luo and C.-K. Tham, "Fairness and social welfare in incentivizing participatory sensing," in *IEEE SECON*, June 2012, pp. 425–433.

[21] C. Cohen, T. Kaplan, and A. Sela, "Optimal rewards in contests," *The RAND Journal of Economics*, vol. 39, no. 2, pp. 434–451, 2008.

[22] T. Kaplan, I. Luski, A. Sela, and D. Wettstein, "All-pay auctions with variable rewards," *Journal of Industrial Economics*, vol. 50, no. 4, pp. 417–430, 2002.

[23] G. Fibich, A. Gavious, and A. Sela, "All-pay auctions with risk-averse players," *Int J Game Theory*, vol. 34, pp. 583–599, 2006.

[24] R. P. McAfee and J. McMillan, "Auctions with a stochastic number of bidders," *Journal of Economic Theory*, no. 43, pp. 1–19, 1987.

[25] D. Levin and E. Ozdenoren, "Auctions with uncertain numbers of bidders," *Journal of Economic Theory*, no. 118, pp. 229–251, 2004.

[26] R. M. Harstad, J. H. Kagel, and D. Levin, "Equilibrium bid functions for auctions with an uncertain number of bidders," *Economics Letters*, no. 33, pp. 35–40, 1990.

[27] M. Haviv and I. Milchtaich, "Auctions with a random number of identical bidders," *Economics Letters*, vol. 114, no. 2, pp. 143–146, 2012.

[28] C.-K. Tham and T. Luo, "Quality of contributed service and market equilibrium for participatory sensing," in *IEEE DCOSS*, May 2013, pp. 133–140.

[29] S. Athey, "Single crossing properties and the existence of pure strategy equilibria in games of incomplete information," *Econometrica*, vol. 69, no. 4, pp. 861–889, 2001.

[30] P. Milgrom and I. Segal, "Envelope theorems for arbitrary choice sets," *Econometrica*, no. 70, pp. 583–601, 2002.

[16] It can be verified that the prize offered by the principal using CDP has an expected value of $n/24$, which induces each agent to contribute a value independent of n (1/6 if averaging over all types) to the revenue, resulting in a profit of $n/8$.