Framework for Delay Analysis of Channel-aware Wireless Schedulers

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Abstract

We consider the scheduling problem over wireless channels for real-time applications where the Quality of Service requirements are given in terms of delay statistics. Although many wireless channel-state dependent (CSD) schedulers have been proposed recently, their contributions lie in the design of the scheduling mechanism to meet some performance objectives. However, the delay performance are specified in terms of first-order statistics, i.e., average or worst-case values, which are insufficient to characterize the scheduler's performance.

In this paper, we develop a framework for the stochastic analysis of the delay performance of CSD schedulers. We derive the delay probability density function and its moments for a Two-State Markov Chain Error Model using a matrix formulation approach. We demonstrate the applicability of our analysis numerically by considering the admissibility of a wireless scheduler in terms of a minimum throughput requirement. This translates to an upper bound on the mean Head-of-Line (HOL) packet delay. Subsequently, we evaluate the buffer size requirement of the wireless receiver and highlight the trade-off between buffer size requirements and channel efficiency.

I. Introduction

A. Related work in Wireless Scheduling

An abundance of scheduling policies that provide guaranteed Quality of Service (QoS) for wireline networks exists in the literature ([1], [2], [3], to name a few). However, direct application of these policies to the wireless media is not useful due to the following unique characteristics: (a) high channel error rate (b) bursty and time-varying channel capacity (c) location dependent channel capacity (d) user mobility and (e) power constraint of mobile users.

The notion of channel-state dependence (CSD) or awareness was introduced in [4] to improve the performance of wireline schedulers when deployed in a wireless media by exploiting characteristics (b) and (c). [5] offers a comprehensive survey of variants of CSD schedulers that differ in the mechanism of choosing the instantaneous 'best' flow to transmit while satisfying different constraints. Such constraints can often be specified in terms of the long-term fraction of time to be allocated to each user (time-fraction requirement).

In [6], the authors defined the scheduling problem as one of maximizing the average system performance. An opportunistic (equivalent to channel-state aware) scheduling policy is proposed that solves the scheduling problem optimally. In addition, the algorithm also improves every user's average performance relative to any non-opportunistic scheduling policy and also takes into account the short-term performance requirements of users. However, it is unclear how the scheme performs in terms of per-flow QoS when handling delay-sensitive traffic.

The concept of 'compensation' was introduced and employed in CSD schedulers proposed in [7], [8], [9], [10], [11], [12] to achieve a tradeoff between channel efficiency and short-term fairness provision. While wireline scheduling is used under error-free conditions, a wireless adaptation scheme is employed when these conditions no longer prevail. Essentially, flow swapping takes place when an allocated flow is unable to transmit in order to maximize channel efficiency. Flows that 'gave up' their allocated slots are subsequently compensated by flows that 'acquired' those additional slots so as to maintain short-term fairness. The performance of these schedulers in terms of throughput, delay and fairness was analyzed and compared in [13].

B. Research Contributions of This Paper

In this paper, we consider the scheduling problem for real-time applications (e.g., streaming and interactive audio/video) whose QoS requirements can be specified in terms of delay statistics. Many wireless schedulers have been proposed in the literature recently whose main contributions lie in the design of the scheduling mechanism to meet various performance objectives. Although delay analysis has been performed for the proposed schedulers, the metrics used are first order, i.e., average and worst-case values, which are inadequate to characterize the scheduler's performance. For example, the receiver buffer requirement depends on both the mean as well as the variance of the inter-arrival time (i.e. the HOL packet delay of the scheduler).

Our focus is to propose a framework for stochastic analysis of the delay performance of CSD schedulers. Our approach is similar to that adopted in [14], where the authors studied the delay performance of a simple ARQ error control strategy for communications over a bursty channel of a *single* flow. In [15], the author investigated the characteristics and traffic effects of variable-rate communication servers. It is shown that if all input connections to a fluctuation-constrained [16] work-conserving server-node are burstiness-constrained [17], deterministic or statistical bounds on queue length and traffic delay in an isolated work-conserving variable-rate server node can be computed as long as the stability criterion is satisfied. However, the scheduling policy considered is not channel-aware since the channel is assumed to be location-independent. This characteristic is considered in the resource allocation problem in [18], where the authors characterized the stability properties of the system and proposed an optimal allocation policy that maximizes throughput and minimizes delay. However, the results are only applicable for an uncorrelated channel, which is an impractical assumption (characteristic (b)).

We adopt a generic CSD scheduling architecture based on the proposed wireless schedulers and derive the delay probability density function (pdf) for a wireless channel model that takes into account characteristics (b) and (c). Such an analysis offers a more complete characterization of the delay performance of wireless schedulers.

The rest of the paper is organized as follows: In Section II, we define the wireless scheduling model considered in our analysis. In Section III, we describe our approach for the delay analysis. Section IV describes the evaluation of the HOL packet delay pdf while Section V considers the special case of uncorrelated channel errors. Numerical results are presented in Section VI, where we consider an application of our analysis. Concluding remarks are presented in Section VII.

II. Description of CSD Scheduler and Channel Model

We consider a K-flow centralized wireless scheduling scenario where channel access to each flow is allocated in terms of fixed-size time slots. We assume that each flow is always backlogged and comprises constant-size packets with transmission time of one slot. Each flow *j* is characterized by the integer parameter r^j, where the ratio $\frac{r^j}{\sum_{m=1}^{K} r^m}$ denotes the time-fraction requirement of flow *j*, i.e., the fraction of resources that should be allocated to flow *j* in the long term.

A. Wireless Channel Model

We use a Two-State Markov Chain to model the error behavior of the wireless channel of each flow, and define c_i^j to be the channel state of flow *j* in slot *i*, where c_i^j = Good or Bad. The state transition diagram of the error model is shown in Fig. 1(a). For each flow *j*, the model is specified in terms of the parameters, p_G^j and p_{corr}^j, which are defined as follows:

$$\begin{aligned} p_G^j &= \text{Steady State Probability of Channel of flow } j \\ &\quad \text{being in Good State} \\ p_{corr}^j &= p_{BG}^j + p_{GB}^j \end{aligned}$$

where

$$\begin{aligned} p_{GB}^j &= Prob(c_i^j = Bad \mid c_{i-1}^j = Good) \\ p_{BG}^j &= Prob(c_i^j = Good \mid c_{i-1}^j = Bad) \end{aligned}$$

Given p_G^j and p_{corr}^j, p_{GB}^j and p_{BG}^j are computed as follows:

$$\begin{aligned} p_{BG}^j &= p_{corr}^j \cdot p_G^j \\ p_{GB}^j &= p_{corr}^j (1 - p_G^j) \end{aligned}$$

The parameter, p_{corr}^j, is inversely proportional to the level of correlation in the error behavior across successive slots for flow *j*; a value close to 0 indicates high correlation while a value of 1.0 represents the special case of uncorrelated errors since

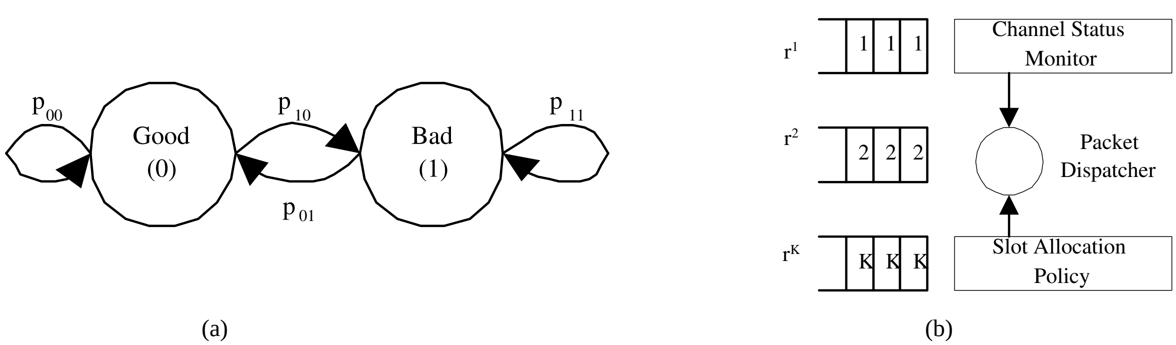


Fig. 1. (a) State Transition Diagram of 2SMC Error Model (b) General Architecture of CSD Wireless Schedulers

$p_{GB}^j = p_{BB}^j = p_B^j$ and $p_{BG}^j = p_{GG}^j = 1 - p_B^j$. We assume that the channel state of each flow is independent of that of any other flow.

In order to characterize the channel process, let us define the state variable x_i^K as the decimal equivalent of the binary representation given by $c_i^K c_i^{K-1} \cdots c_i^1$, where the binary representations of *Good* and *Bad* states are 0 and 1 respectively, as shown in Fig. 1. This is illustrated below for the case of K=2.

$c_i^2 c_i^1$	x_i^2
00	0
01	1
10	2
11	3

The state space of the channel process is given by the set $\{0, 1, 2, \cdots 2^K - 1\}$. Therefore, the state transition probability matrix, $\underline{\underline{A}}_K$*, is of dimensions $2^K \times 2^K$ and can be computed, for $K \geq 2$, using the following recurrence relation:

$$\underline{\underline{A}}_K = \begin{bmatrix} \underline{\underline{A}}_{K-1} \cdot p_{00}^K & \underline{\underline{A}}_{K-1} \cdot p_{01}^K \\ \underline{\underline{A}}_{K-1} \cdot p_{10}^K & \underline{\underline{A}}_{K-1} \cdot p_{11}^K \end{bmatrix}$$

where

$$\underline{\underline{A}}_1 = \begin{bmatrix} p_{00}^1 & p_{01}^1 \\ p_{10}^1 & p_{11}^1 \end{bmatrix}$$

If the row vector $\underline{f}$(i) denotes the pdf of x_i^K, i.e.,

$$\underline{f}(i) = [Prob(x_i^K = x)]_{x=0}^{2^K-1}$$

then,

$$\underline{f}(i) = \underline{f}(i-1) \times \underline{\underline{A}}_K \tag{1}$$

B. CSD Scheduling Model

The CSD scheduling architecture comprises a slot allocation policy (SAP), a channel status monitor (CSM) and a packet dispatcher, as depicted in Fig. 1(b) [4].

At the beginning of each slot *i*, the SAP determines the flow *j* that has the highest priority to transmit in slot *i* according to r^f of each flow *f* using a Weighted-Round Robin (WRR) policy. The CSM maintains the history of $x_{i-m}^K,\ m > 0$ and uses this information to predict x_i^K. We assume, in this analysis, that channel prediction is perfect. If c_i^j = Good, the packet dispatcher will transmit the HOL packet of flow *j*; otherwise, an *arbitration scheme* will be used to select an alternative flow to transmit based on x_i^K.

III. Notion of Constrained State Transition Matrix

Let E_j^f denote the transmission event of flow *f* in a slot that is allocated to flow *j*, where E $\in \{S$,F$\}$ is used to denote a **S**uccessful and de**F**erred transmission respectively. For a given arbitration scheme, the values of x_i^K and *j* determine the probability of occurrence of E_j^f in slot *i*. Conversely stated, given the value of *j*, the occurrence of E_j^f in slot *i* imposes a constraint on the pdf of x_i^K. We define the *constrained state transition matrix* for event E_j^f as follows:

$$\underline{\underline{E}}_j^f = \underline{\underline{A}}_K \times \underline{\underline{M}}(E_j^f)$$

where $\underline{\underline{M}}(E_j^f)$ is a diagonal matrix such that the diagonal element of row *m* is the probability that E_j^f will occur if x_i^K=*m*-1. Therefore,

$$\underline{\underline{S}}_j^f = \underline{\underline{A}}_K \times \underline{\underline{M}}(S_j^f)$$
$$\underline{\underline{F}}_j^f = \underline{\underline{A}}_K \times \underline{\underline{M}}(F_j^f)$$

We note the following:

$$\begin{aligned} \underline{\underline{S}}_j^f + \underline{\underline{F}}_j^f &= \underline{\underline{A}}_K \times \underline{\underline{M}}(S_j^f) + \underline{\underline{A}}_K \times \underline{\underline{M}}(F_j^f) \\ &= \underline{\underline{A}}_K \times \underline{\underline{M}}(S_j^f \cup F_j^f) \\ &= \underline{\underline{A}}_K \end{aligned}$$

*Note that the notations $\underline{\mathrm{A}}$ and $\underline{\underline{\mathrm{A}}}$ denote vector and matrix A respectively. The notations . and $\times$ denote scalar and matrix products respectively.

Hence, $\underline{\underline{F}}_j^f$ can be expressed in terms of $\underline{\underline{S}}_j^f$ and $\underline{\underline{A}}_K$.

If we replace $\underline{\underline{A}}_K$ in Eq. (1) by $\underline{\underline{E}}_j^f$, we have the following:

$$\begin{aligned}\underline{f}(i) &= \underline{f}(i-1) \times \underline{\underline{E}}_j^f \\ &= [\text{Prob}(x_i^K = x,\ E_j^f \text{occurs in slot } i)]_{x=0}^{2^K-1}\end{aligned}$$

Similarly,

$$\begin{aligned}\underline{f}(i+1) &= \underline{f}(i-1) \times \underline{\underline{E}}_j^f \times \underline{\underline{E}}_p^f \\ &= [\text{Prob}(x_{i+1}^K = x,\ E_j^f, E_p^f \text{ occur in slots } i \text{ and } i+1 \text{ respectively})]_{x=0}^{2^K-1}\end{aligned}$$

In general, if E(u) is the event in slot *u* and $\bigcap_{u=i}^{i+n}$E(u) denotes the sequence of events {E(i),E(i+1),$\cdots$,E(i+n)} in slots [i:i+n], then we can define a *joint constrained state transition matrix*, $\prod_{u=i}^{i+n}\underline{\underline{E}}$(u), of $\bigcap_{u=i}^{i+n}$E(u) such that:[†]

$$\begin{aligned}\underline{f}(i+n) &= \underline{f}(i-1) \times \prod_{u=i}^{i+n} \underline{\underline{E}}(u) \\ &= [\text{Prob}(x_i^K = x,\ \bigcap_{u=i}^{i+n} E(u) \text{ occurs})]_{x=0}^{2^K-1}\end{aligned}$$

Therefore,

$$\begin{aligned}\text{Prob}(\bigcap_{u=i}^{i+n} E(u) \text{ occurs}) &= \sum_{x=0}^{2^K-1} \text{Prob}(x_i^K = x,\ \bigcap_{u=i}^{i+n} E(u) \text{ occurs}) \\ &= \underline{f}(i+n) \times \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \qquad (2) \\ &= \underline{f}(i-1) \times \prod_{u=i}^{i+n} \underline{\underline{E}}(u) \times \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}\end{aligned}$$

Example 1: Consider a 2-flow CSD scheduler where r^1=r^2, and assume that a flow 1 packet becomes HOL at the beginning of slot *i* that is allocated to flow 2. We would like to determine the probability that this packet will be transmitted in slot *i*+1, i.e., Prob(HOL delay = 2 slots).

Since r^1=r^2, the SAP allocates slots alternately to each flow and therefore slot *i*+1 is allocated to flow 1. For the packet to be transmitted in slot *i*+1, the events F_2^1 and S_1^1 must occur in slots *i* and *i*+1 respectively. According to the CSD scheduling mechanism, F_2^1 occurs in slot *i* if x_i^2={0,1,3}. Therefore,

$$\underline{\underline{F}}_2^1 = \underline{\underline{A}}_2 \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Similarly, S_1^1 occurs in slot *i*+1 if x_{i+1}^2={0,2}. Therefore,

$$\underline{\underline{S}}_1^1 = \underline{\underline{A}}_2 \times \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Hence,

$$\text{Prob(HOL delay = 2 slots)} = \underline{f}(i-1) \times \underline{\underline{F}}_2^1 \times \underline{\underline{S}}_1^1 \times \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}$$

[†]Note that the notation $\prod_a^b$ when used with matrices refers to a sequence of matrix products where the order is important and given by a,a+1,a+2,$\cdots$ b.

IV. Evaluation of HOL packet delay pdf

In our earlier work [19], we have developed a Markov model for a K-flow CSD scheduler in order to analyze its QoS performance for the Two-State Markov Chain error model. We have assumed *homogeneity* in both rate and channel, i.e., r^j=1 and (p_G^j, p_{corr}^j) = (p_G, p_{corr}) respectively for $1 \leq j \leq K$. We considered a uniform arbitration scheme, i.e., when an allocated flow fails to transmit, a flow is *randomly* chosen to transmit amongst those with good channels. The scheduler model has been shown to be ergodic, and therefore its delay pdf under steady-state conditions exists. We have numerically evaluated the delay pdf for small values of K.

In this section, we extend the analysis by considering different arbitration schemes as well as channel heterogeneity. Let d^f(n) denote the pdf of the HOL packet delay *n* for flow *f* under steady-state conditions. Since all flows are always backlogged, *n* corresponds to the duration between two successive flow *f* transmissions. Let us assume that S_j^f occurs in the first transmission, and denote the sequence of subsequent events that must occur up to the second transmission by $D_j^f(n)$, as illustrated in Fig. 2. We note that between two successive S events, there can only be F events.

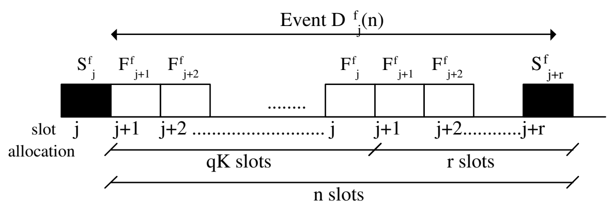


Fig. 2. Events between two successive transmissions of flow *f*

Then, as in Eq. (2), d^f(n) can be expressed as follows:

$$\begin{aligned} d^f(n) &= \sum_{j=1}^{K} \text{Prob}(D_j^f(n)) \\ &= \sum_{j=1}^{K} \underline{f}_j^f \times \underline{\underline{D}}_j^f(n) \times \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \end{aligned} \quad (3)$$

where $\underline{f}_j^f$ = [Prob($x^K = x$, S_j^f occurs)]$_{x=0}^{2^K-1}$ and $\underline{\underline{D}}_j^f(n)$ is the joint constrained state transition matrix of D_j^f(n) given as follows:

$$\underline{\underline{D}}_j^f(n) = [\underline{\underline{C}}_j^f]^q \times [\prod_{m=j+1}^{j+r-1} \underline{\underline{F}}_m^f] \times \underline{\underline{S}}_{j+r}^f \quad (4)$$

where $\underline{\underline{C}}_j^f = \prod_{m=j+1}^{j+K} \underline{\underline{F}}_m^f$, q = $\frac{n-r}{K}$, and *r* is given as follows:

$$r = \begin{cases} \text{n modulo K}, & \text{n modulo K} > 0; \\ K, & otherwise. \end{cases}$$

We drop the slot index *i* in the notations $\underline{f}(i)$ and x_i^K since *i* always refers to a slot where a flow *f* transmission take place. In addition, we note that all subscripts corresponding to flow indices for all the matrix notations are modulo K.

From Eq. (3), d^f(n) can be expressed in terms of $\{\underline{\underline{S}}_j^f,\ \underline{f}_j^f\}_{j=1}^K$. The evaluation of these terms are described next.

A. Evaluation of $\underline{f}_j^f$

Since the SAP is independent of the channel process, packet *m* of flow *f* becomes HOL with a flow *j*+1 allocation if packet *m*-1 is transmitted in a slot allocated to flow *j*. Hence, under steady-state conditions, $\underline{f}_j^f$ can be evaluated based on a recurrence relation in terms of $\{\underline{f}_j^f\}_{j=1}^K$ and $\{\underline{\underline{D}}_j^f(n)\}_{j=1}^K$. We begin with the evaluation of $\underline{f}_1^f$ as follows:

$$\begin{aligned} \underline{f}_1^f &= \underline{f}_1^f \times \sum_{q=1}^{\infty} \underline{\underline{D}}_1^f(q \cdot K) + \underline{f}_2^f \times \sum_{q=1}^{\infty} \underline{\underline{D}}_2^f(q \cdot K - 1) + \cdots + \underline{f}_K^f \times \sum_{q=1}^{\infty} \underline{\underline{D}}_K^f(q \cdot K - [K-1]) \\ &= \sum_{t=1}^{K} \sum_{q=1}^{\infty} \underline{f}_t^f \times \underline{\underline{D}}_t^f(q \cdot K - [t-1]) \end{aligned} \quad (5)$$

Substituting Eq. (4) into Eq. (5) and simplifying, we obtain the following equation for $\underline{f}_1^f$:

$$\underline{f}_1^f = \sum_{t=1}^{K} \underline{f}_t^f \times (\underline{\underline{I}} - \underline{\underline{C}}_t^f)^{-1} \times \prod_{m=t+1}^{K} \underline{\underline{F}}_m^f \times \underline{\underline{S}}_1^f$$

In general, the recurrence equations for $\underline{f}_j^f, 1 \leq j \leq K$, can be written as follows:

$$\underline{f}_j^f = [\sum_{t=1}^{j-1} \underline{f}_t^f \times (\underline{\underline{I}} - \underline{\underline{C}}_t^f)^{-1} \times \prod_{m=t+1}^{j-1} \underline{\underline{F}}_m^f + \sum_{t=j}^{K} \underline{f}_t^f \times (\underline{\underline{I}} - \underline{\underline{C}}_t^f)^{-1} \times \prod_{m=t+1}^{K+j-1} \underline{\underline{F}}_m^f] \times \underline{\underline{S}}_j^f \tag{6}$$

We can express Eq. (6) in matrix form as follows:

$$\underline{f} = \underline{f} \times \underline{\underline{InvC}} \times \underline{\underline{FS}}$$

where

$$\underline{f} = \begin{bmatrix} \underline{f}_K^f & \underline{f}_{K-1}^f & \cdots & \underline{f}_1^f \end{bmatrix}$$

$$\underline{\underline{InvC}} = \begin{bmatrix} (\underline{\underline{I}} - \underline{\underline{C}}_K^f)^{-1} & 0 & \cdots & 0 \\ 0 & (\underline{\underline{I}} - \underline{\underline{C}}_{K-1}^f)^{-1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & (\underline{\underline{I}} - \underline{\underline{C}}_1^f)^{-1} \end{bmatrix}$$

$$\underline{\underline{FS}} = \begin{bmatrix} \prod_{m=1}^{K-1} \underline{\underline{F}}_m^f \times \underline{\underline{S}}_K^f & \cdots & \underline{\underline{F}}_1^f \times \underline{\underline{S}}_2^f & \underline{\underline{S}}_1^f \\ \underline{\underline{S}}_K^f & \cdots & \underline{\underline{F}}_K^f \times \underline{\underline{F}}_1^f \times \underline{\underline{S}}_2^f & \underline{\underline{F}}_K^f \times \underline{\underline{S}}_1^f \\ \vdots & \vdots & \vdots & \vdots \\ \prod_{m=2}^{K-1} \underline{\underline{F}}_m^f \times \underline{\underline{S}}_K^f & \cdots & \underline{\underline{S}}_2^f & \underline{\underline{F}}_2^f \cdots \underline{\underline{F}}_K^f \times \underline{\underline{S}}_1^f \end{bmatrix}$$

B. Evaluation of $\underline{\underline{S}}_j^f$

We recall that $\underline{\underline{S}}_j^f$ is defined in terms of $\underline{\underline{A}}_K$ and a diagonal matrix, $\underline{\underline{M}}(S_j^f)$, where the diagonal element of row *m* is the probability that S_j^f will occur in a slot where x^K=*m*-1. Let $\underline{M}(S_j^f)$ be a row vector comprising the diagonal elements of $\underline{\underline{M}}(S_j^f)$, i.e.,

$$\underline{M}(S_j^f) = [\text{Prob}(S_j^f \text{ occurs} \mid x^K = x)]_{x=0}^{2^K-1}$$

The evaluation of $\underline{M}(S_j^f)$ depends on the arbitration scheme used by the CSD scheduler, which determines the alternative flow to transmit in the event that an allocated flow is unable to transmit. We consider two arbitration schemes: (a) *uniform* and (b) *prioritized* arbitration.

1) Uniform Arbitration (CSD_{UA} Scheduler): With uniform arbitration, a flow is *randomly* selected amongst those that currently perceive good channels when an allocated flow is unable to transmit.

To determine the vector $\underline{M}(S_j^f)$, we first initialize it to a vector of zeros. For each x^K, the corresponding value of c^j for 1$\leq j \leq K$ (denoted $c^j(x^K)$) is given as follows:

$$c^j(x^K) = \begin{cases} 0, & \lceil \frac{x^K}{2^{j-1}} \rceil modulo\ 2 = 1; \\ 1, & otherwise. \end{cases} \tag{7}$$

The necessary conditions for S_j^f to occur are given as follows:

$$\begin{aligned} c^f(x^K) &= 0 \\ c^j(x^K) &= 1 \end{aligned} \tag{8}$$

Based on Eq. (7), we can determine the range of x^K, $Index_{x^K}$, for which Eq. (8) are satisfied. For each $x^K \ \epsilon \ Index_{x^K}$, we determine the total number of flows (other than flow *f*) contending for transmission, *Total*, i.e., total number of flows *flow*, such that $c^{flow}(x^K)$=0. Since there are altogether *Total* + 1 flows that are eligible for transmission, the probability that any flow is selected for transmission is $\frac{1}{Total+1}$. Hence, the corresponding entry in $\underline{M}(S_j^f)$ is $\frac{1}{Total+1}$.

2) Prioritized Arbitration ($CSD_{PA}(P_h)$ Scheduler): An inherent characteristic of CSD scheduling is that flows are able to transmit in other slots in addition to those allocated to them. Although this results in improvement in channel efficiency, the delay variation may be increased, giving rise to larger receiver buffer requirements, which is undesirable. With uniform arbitration, it is possible that a flow may transmit in two successive slots (including the slot allocated to it) and then remain silent for about K slots until the next transmission in its allocated slot. This introduces a large delay variation, which can be reduced by using prioritized arbitration. The motivation behind this arbitration scheme is that when an allocated flow is unable to transmit, *preference* for transmission is given to flows whose next allocation is as *far* away as possible from the given slot. In this way, the delay variance can be reduced with respect to that achievable with uniform arbitration.

Quantitatively, let $Flow_P^j$ modulo K be the set of flows with priority level P, where 0$\leq P \leq P_{max}$, in a slot allocated to flow *j*. For each *j*, 1$\leq j \leq K$, if a *smaller* P denotes a *higher* priority of transmission, then $Flow_P^j$ is defined as follows:

$$Flow_P^j = \begin{cases} j, & P=0; \\ \{j-P_{max}+P-1, j+P_{max}-P+1\}, & 1 \leq P \leq P_{max}. \end{cases}$$

where

$$P_{max} = \begin{cases} \frac{K-1}{2}, & Kodd; \\ \frac{K}{2}, & Keven. \end{cases}$$

Hence, in a slot allocated to flow *j*, flow *j* has the highest priority to transmit. In the event that it defers its transmission, uniform arbitration is used to determine which of the flow(s) $\in Flow_1^j$ will transmit. If none of the flows can transmit (i.e., all their channels are in bad state), we descend to $Flow_2^j$ and so on.

We can parametize the prioritized arbitration scheme such that in any slot allocated to flow *j*, only flows $\in \{Flow_P^j\}_{P=0}^{P_h}$ are allowed to transmit. Hence, if $f \notin \{Flow_P^j\}_{P=0}^{P_h}$, then flow f will not be allowed to transmit. Hence, the choice of P_h may represent a tradeoff between channel efficiency and delay variance under good channel conditions : a larger P_h implies more opportunities for flow f to transmit which may in turn result in a larger delay variance.

To determine the vector $\underline{M}(S_j^f)$, we first determine the priority level of flow *f*, Pr_f, using Eq. (9). If $Pr_f > P_h$, then flow f is unable to transmit in the given slot and we set $\underline{M}(S_j^f)$ to a vector of zeros. Otherwise, for each x^K, for S_j^f to occur, in addition to Eq. (8), all flows with priority $< Pr_f$ must have erroneous channels. The latter condition can be expressed as follows:

$$c^{flow}(x^K) = 1 \quad \forall \; flow \in \{Flow_P^j\}_{P=1}^{Pr_f-1} \tag{9}$$

Based on Eq. (7), we can determine the range of x^K, $Index_{x^K}$, such that Eq. (8) and Eq. (9) are satisfied.

Next, we determine the flow *fl* that has the same priority level as flow *f* using Eq. (9). For each $x^K \epsilon \; Index_{x^K}$, if $c^{fl}(x^K)$=0, then we have two contending flows (including flow *f*) and the entry in $\underline{M}(S_j^f)$ = $\frac{1}{2}$; otherwise, flow f is the only eligible flow to transmit and therefore the entry in $\underline{M}(S_j^f)$=1.

V. Special Case: Uncorrelated channel errors

In Section IV, we developed a matrix formulation to determine the pdf of the HOL packet delay *n* for an arbitrary flow *f* in terms of the channel parameters as well as the arbitration scheme employed by the CSD scheduler. In this section, we will illustrate the derivation of moments of *n* under the following assumptions for 1$\leq j \leq$ K:

(a) channel errors are uncorrelated, i.e., p_{corr}^j=1.0 and

(b) channel process is homogeneous, i.e., (p_G^j,p_{corr}^j) = (p_G,p_{corr}).

Assumption (a) implies that the matrices defined in Section IV collapse into scalar quantities. Hence, we will drop $=$ from these notations for this section. Together with rate-homogeneity, assumption (b) implies that the scheduling system is homogeneous and hence, the delay performance of all flows are identical. Therefore, we drop the superscript *f* and consider only the performance of flow 1.

Since F_m is a scalar, we note that C_j = C = $\prod_{m=1}^{K} F_m$, 1$\leq j \leq K$. Hence, the corresponding matrix form for Eq. (6) is given as follows:

$$\underline{f} = \underline{f} \times \underline{\underline{InvC}} \times \underline{\underline{FS}} \tag{10}$$

where

$$\begin{aligned}
\underline{f} &= \begin{bmatrix} f_K & f_{K-1} & \cdots & f_1 \end{bmatrix} \\
\underline{\underline{InvC}} &= \frac{1}{1-C}\begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix} \\
\underline{\underline{FS}} &= \begin{bmatrix} \prod_{m=1}^{K-1} F_m \cdot S_K & \cdots & F_1 \cdot S_2 & S_1 \\ S_K & \cdots & F_K \cdot F_1 \cdot S_2 & F_K \cdot S_1 \\ \vdots & \vdots & \vdots & \vdots \\ \prod_{m=2}^{K-1} F_m \cdot S_K & \cdots & S_2 & F_2 \cdot \cdot F_K \cdot S_1 \end{bmatrix}
\end{aligned}$$

We have K-1 independent equations in Eq. (10) to solve for K unknowns. The sum of probability, $\sum_{j=1}^{K} f_j$ = 1, offers the additional equation needed to solve for f_j. We notice that in Eq. (10), for each equation with f_j on the LHS, there is a common factor, S_j, on the RHS. Hence, a good guess for the solution of f_j is $f_j = \frac{S_j}{CONST}$. According to the sum of probability, we have CONST = $\sum_{f=1}^{K} S_f$ (henceforth, denoted Σ_S).

Lemma 1: For uncorrelated channel errors, the probability that a flow 1 packet transmits in a slot allocated to flow *j*, f_j, for a homogeneous K-flow CSD scheduler is given by:

$$f_j = \frac{S_j}{\sum_{f=1}^{K} S_f} = \frac{S_j}{\Sigma_S}$$

Proof: Substituting the expression for f_j into the RHS of Eq. (10), we have the following:

$$RHS = \frac{S_j}{\Sigma_S(1-C)}[\sum_{u=1}^{j-1} S_u \prod_{m=u+1}^{j-1} F_m + \sum_{u=j}^{K} S_u \prod_{m=u+1}^{K+j-1} F_m]$$

Expanding the terms within the brackets (denoted Σ), and writing them in decreasing order of the number of elements within each term, we obtain the following summation:

$$\begin{aligned}
\Sigma &= S_j \cdot F_{j+1} \cdot F_{j+2} \cdots F_K \cdot F_1 \cdots F_{j-1} \\
&+ S_{j+1} \cdot F_{j+2} \cdot F_{j+3} \cdots F_{j-1} \\
&\vdots \\
&+ S_{j-2} \cdot F_{j-1} \\
&+ S_{j-1}
\end{aligned}$$

Adding C = $\prod_{f=1}^{K} F_f = F_j \cdot F_{j+1} \cdots F_K \cdot F_1 \cdots F_{j-1}$ to the first term of the RHS of Σ, we obtain $F_{j+1} \cdot F_{j+2} \cdots F_K \cdot F_1 \cdots F_{j-1}$. Adding this to the second term, we obtain $F_{j+2} \cdot F_{j+3} \cdots F_K \cdot F_1 \cdots F_{j-1}$. Proceeding in the same manner, eventually, we obtain

$$\Sigma + C = 1$$

Therefore, we have the following:

$$\begin{aligned}
RHS &= \frac{S_j}{\Sigma_S(1-C)}(1-C) \\
&= \frac{S_j}{\Sigma_S} \\
&= LHS
\end{aligned}$$

■

If we write $n = q \cdot K + r$, where $q \geq 0$, $1 \leq r \leq$ K, then from Eq. (3) and Lemma 1, we have the following:

Theorem 1: For uncorrelated channel errors, the pdf of the HOL packet delay of any flow for a homogeneous K-flow CSD-scheduler is given as follows:

$$\begin{aligned} d(q\cdot K+r) &= \frac{C^q}{\Sigma_S}\sum_{j=1}^{K} S_j\cdot S_{j+r}\prod_{m=j+1}^{j+r-1}F_m \\ where \quad & \quad q\geq 0,\ 1\leq r\leq K \end{aligned}$$

A. Evaluation of the moments of the HOL packet delay, n

In this section, we shall obtain expressions for the first and second moment of *n*.

Theorem 2: For uncorrelated channel errors, the expected HOL packet delay for a homogeneous K-flow CSD scheduler is given as follows:

$$E[n]=\frac{K}{\Sigma_S}$$

Proof: Beginning with the definition of E[n] and using Theorem 1, we have the following:

$$\begin{aligned} E[n] &= \sum_{q=0}^{\infty}\sum_{r=1}^{K}(q\cdot K+r).d(q\cdot K+r) \\ &= \sum_{q=0}^{\infty} q\cdot K\sum_{r=1}^{k} d(q\cdot K+r)+\sum_{q=0}^{\infty}\sum_{r=1}^{K} r\cdot d(q\cdot K+r) \\ &= \sum_{q=0}^{\infty}\frac{q\cdot K\cdot C^q}{\Sigma_S}\sum_{r=1}^{K}\sum_{j=1}^{K} S_j\cdot S_{j+r}\prod_{m=j+1}^{j+r-1}F_m+\sum_{q=0}^{\infty}\frac{C^q}{\Sigma_S}\sum_{r=1}^{K} r\sum_{j=1}^{K} S_j\cdot S_{j+r}\prod_{m=j+1}^{j+r-1}F_m \\ &= \frac{K\cdot C}{\Sigma_S(1-C)^2}\Sigma_0+\frac{1}{\Sigma_S(1-C)}\Sigma_1 \end{aligned} \tag{11}$$

where

$$\Sigma_i = \sum_{r=1}^{K} r^i\sum_{j=1}^{K} S_j\cdot S_{j+r}\prod_{m=j+1}^{j+r-1}F_m$$

Using the fact that sum of probabilities is 1, we have $\sum_{q=0}^{\infty}\sum_{r=1}^{K} d(q\cdot K+r)$=1. Substituting for $D^{q.K+r}$, we have the following:

$$\begin{aligned} \sum_{q=0}^{\infty}\frac{C^q}{\Sigma_S}\Sigma_0 &= 1 \\ \Rightarrow \frac{1}{\Sigma_S(1-C)}\Sigma_0 &= 1 \\ \Rightarrow \Sigma_0 &= \Sigma_S(1-C) \end{aligned}$$

Σ_1 can be expanded as follows:

$$\begin{array}{llll} S_1\cdot S_2 & +S_2\cdot S_3 & \cdots & +S_K\cdot S_1 \\ +2[S_1\cdot F_2\cdot S_3 & +S_2\cdot F_3\cdot S_4 & \cdots & +S_K\cdot F_1\cdot S_2] \\ +3[S_1\cdot F_2\cdot F_3\cdot S_4 & +S_2\cdot F_3\cdot F_4\cdot S_5 & \cdots & +S_K\cdot F_1\cdot F_2\cdot S_3] \\ \vdots & \vdots & \vdots & \vdots \\ +K[S_1\cdot F_2\cdot\cdot F_K\cdot S_1 & +S_2\cdot F_3\cdot\cdot F_1\cdot S_2 & \cdots & +S_K\cdot F_1\cdot\cdot F_{K-1}\cdot S_K] \end{array}$$

Let us consider the elements of the first column of Σ_1. Excluding the common factor, S_1, the summation can be written alternatively as follows:

$$\begin{array}{llllll} K\{S_2 & +F_2\cdot S_3 & \cdots & +F_2\cdots F_{K-1}\cdot S_K & +F_2\cdots F_K\cdot S_1\} \\ -\{S_2 & +F_2\cdot S_3 & \cdots & +F_2\cdots F_{K-1}\cdot S_K & \} \\ \vdots & \vdots & \vdots & \vdots & \\ -\{S_2 & +F_2\cdot S_3 & & & \} \\ -\{S_2 & & & & \} \end{array}$$

The above summation can be simplified to the following expression:

$$K(1-C)-\sum_{j=2}^{K}(1-\prod_{m=2}^{j}F_m)=1+\sum_{j=2}^{K}\prod_{m=2}^{j}F_m-K\cdot C$$

Similarly, excluding the common factor, S_2, the summation of the elements of the second column of Σ_1 can be expressed as the following expression:

$$1 + \sum_{j=3}^{K+1} \prod_{m=3}^{j} F_m - K \cdot C$$

Proceeding in the same manner for the remaining columns, we obtain the following expression for Σ_1:

$$\Sigma_1 = \sum_{u=1}^{K} [1 + \sum_{j=u+1}^{K+u-1} \prod_{m=u+1}^{j} F_m - K \cdot C] S_u$$

Substituting for S_j = 1-F_j, and after cancellation of common terms and simplification, we have the following:

$$\Sigma_1 = K(1-C) - \Sigma_S K \cdot C$$

Substituting for Σ_0 and Σ_1 in Eq. (11), we have the following:

$$\begin{aligned} E[n] &= \frac{K \cdot C \Sigma_S (1-C)}{\Sigma_S (1-C)^2} + \frac{K(1-C) - \Sigma_S K \cdot C}{\Sigma_S (1-C)} \\ &= \frac{K \cdot C}{1-C} + \frac{K}{\Sigma_S} - \frac{K \cdot C}{1-C} \\ &= \frac{K}{\Sigma_S} \end{aligned}$$

■

Remark 1: The expression for E[n] can also be derived intuitively. We begin by giving a qualitative interpretation of Σ_S. Since S_j denotes the probability of flow 1 transmission in a slot allocated to flow *j*, Σ_S denotes the probability that flow 1 will transmit (at all) within a period of K slots. Hence, $\frac{\Sigma_S}{K}$ denotes the probability that flow 1 transmits in *any* slot, which is equivalent to its throughput. Since the mean HOL packet delay and throughput are reciprocals of each other, we have E[n] = $\frac{1}{throughput}$ = $\frac{K}{\Sigma_S}$.

Theorem 3: For uncorrelated channel errors, the second moment of the HOL packet delay for a homogeneous K-flow CSD scheduler is given as follows:

$$E[n^2] = \frac{K(1+C) + 2\sum_{r=1}^{K} \sum_{i=1}^{K-1} \prod_{m=r}^{r+i-1} F_m}{(1-C)\Sigma_S}$$

Proof: Beginning with the definition of E[n^2] and using Theorem 1, we have the following:

$$\begin{aligned} E[n^2] &= \sum_{q=0}^{\infty} \sum_{r=1}^{K} (q \cdot K + r)^2 d(q \cdot K + r) \qquad (12) \\ &= \sum_{q=0}^{\infty} \sum_{r=1}^{K} (q^2 K^2 + r^2 + 2q \cdot K \cdot r) C^q \sum_{j=1}^{K} S_j \cdot S_{j+r} \prod_{m=j+1}^{j+r-1} F_m \\ &= \frac{1}{\Sigma_S} [\sum_{q=0}^{\infty} q^2 K^2 C^q \Sigma_0 + \sum_{q=0}^{\infty} C^q 2q \cdot K \Sigma_1 + \sum_{q=0}^{\infty} C^q \sum_{r=1}^{K} r^2 \Sigma_2] \\ &= \frac{1}{\Sigma_S} [\frac{K^2 C(1+C)}{(1-C)^3} \Sigma_0 + \frac{2K \cdot C}{(1-C)^2} \Sigma_1 + \frac{1}{(1-C)} \Sigma_2] \\ &= \frac{K^2 C}{1-C} + \frac{2K^2 C}{(1-C)\Sigma_S} + \frac{1}{\Sigma_S (1-C)} \Sigma_2 \end{aligned}$$

In a manner similar to Σ_1, Σ_2 can be written as follows:

$$\begin{array}{llll} S_1 \cdot S_2 & +S_2 \cdot S_3 & \cdots & +S_K \cdot S_1 \\ +2^2[S_1 \cdot F_2 \cdot S_3 & +S_2 \cdot F_3 \cdot S_4 & \cdots & +S_K \cdot F_1 \cdot S_2] \\ +3^2[S_1 \cdot F_2 \cdot F_3 \cdot S_4 & +S_2 \cdot F_3 \cdot F_4 \cdot S_5 & \cdots & +S_K \cdot F_1 \cdot F_2 \cdot S_3] \\ \vdots & \vdots & \vdots & \vdots \\ +K^2[S_1 \cdot F_2 \cdot \cdot F_K \cdot S_1 & +S_2 \cdot F_3 \cdot \cdot F_1 \cdot S_2 & \cdots & +S_K \cdot F_1 \cdot \cdot F_{K-1} \cdot S_K] \end{array}$$

Let us consider the elements of the first column of Σ_2. Excluding the common factor, S_1, the summation can be written alternatively as follows:

$$\begin{array}{rllllll} K^2\{S_2 & +F_2\cdot S_3 & \cdots & +F_2\cdot F_3\cdots F_{K-2}\cdot S_{K-1} & +F_2\cdot F_3\cdots F_{K-1}\cdot S_K & +F_2\cdot F_3\cdots F_K\cdot S_1\} & - \\ (2K-1)\{S_2 & +F_2\cdot S_3 & \cdots & +F_2\cdot F_3\cdots F_{K-2}\cdot S_{K-1} & +F_2\cdot F_3\cdots F_{K-1}\cdot S_K & \} & - \\ (2K-3)\{S_2 & +F_2\cdot S_3 & \cdots & +F_2\cdot F_3\cdots F_{K-2}\cdot S_{K-1} & & \} & - \\ & & \vdots & & & & - \\ 5\{S_2 & +F_2\cdot S_3 & & & & \} & - \\ 3\{S_2 & & & & & \} & \end{array}$$

The above summation can be simplified as follows:

$$\begin{array}{rl} & K^2(1-C)-(2K-1)(1-F_2\cdot F_3\cdots F_K)-(2K-3)(1-F_2\cdot F_3\cdots F_{K-1})-\cdots-5(1-F_2\cdot F_3)-3(1-F_2) \\ = & K^2(1-C)-\sum_{r=0}^{K-1}(2r+1)+\{1+3F_2+5F_2\cdot F_3+\cdots+(2K-1)(F_2\cdot F_3\cdots F_K)\} \\ = & \begin{array}{lllllll} -K^2C+1 & +F_2 & +F_2\cdot F_3 & \cdots & +F_2\cdot F_3\cdots F_{K-1} & +F_2\cdot F_3\cdots F_K \\ & +2(F_2 & +F_2\cdot F_3 & \cdots & +F_2\cdot F_3\cdots F_{K-1} & +F_2\cdot F_3\cdots F_K) \\ & & +2(F_2\cdot F_3 & \cdots & +F_2\cdot F_3\cdots F_{K-1} & +F_2\cdot F_3\cdots F_K) \\ & & & \vdots & & \\ & & & & +2(F_2\cdot F_3\cdots F_{K-1} & +F_2\cdot F_3\cdots F_K) \\ & & & & & +2F_2\cdot F_3\cdots F_K \end{array} \end{array}$$

Taking into consideration the common factor, S_1=1-F_1, we obtain the following:

$$\begin{array}{llllll} -K^2C\cdot S_1+1 & +F_2 & +F_2\cdot F_3 & \cdots & +F_2\cdot F_3\cdots F_{K-1} & +F_2\cdot F_3\cdots F_K \\ & +2(F_2 & +F_2\cdot F_3 & \cdots & +F_2\cdot F_3\cdots F_{K-1} & +F_2\cdot F_3\cdots F_K) \\ & & +2(F_2\cdot F_3 & \cdots & +F_2\cdot F_3\cdots F_{K-1} & +F_2\cdot F_3\cdots F_K) \\ & & & \vdots & & \\ & & & & +2(F_2\cdot F_3\cdots F_{K-1} & +F_2\cdot F_3\cdots F_K) \\ & & & & & +2F_2\cdot F_3\cdots F_K \\ -F_1 & -F_1\cdot F_2 & -F_1\cdot F_2\cdot F_3 & \cdots & -F_1\cdot F_2\cdot F_3\cdots F_{K-1} & -F_1\cdot F_2\cdot F_3\cdots F_K \\ & -2(F_1\cdot F_2 & +F_1\cdot F_2\cdot F_3 & \cdots & +F_1\cdot F_2\cdot F_3\cdots F_{K-1} & +F_1\cdot F_2\cdot F_3\cdots F_K) \\ & & -2(F_1\cdot F_2\cdot F_3 & \cdots & +F_1\cdot F_2\cdot F_3\cdots F_{K-1} & +F_1\cdot F_2\cdot F_3\cdots F_K) \\ & & & \vdots & & \\ & & & & -2(F_1\cdot F_2\cdot F_3\cdots F_{K-1} & +F_1\cdot F_2\cdot F_3\cdots F_K) \\ & & & & & -2F_1\cdot F_2\cdot F_3\cdots F_K \end{array}$$

In a similar manner, the elements of the second column of Σ_2 can be expressed as follows:

$$\begin{array}{llllll} -K^2C\cdot S_2+1 & +F_3 & +F_3\cdot F_4 & \cdots & +F_3\cdot F_4\cdots F_K & +F_3\cdot F_4\cdots F_1 \\ & +2(F_3 & +F_3\cdot F_4 & \cdots & +F_3\cdot F_4\cdots F_K & +F_3\cdot F_4\cdots F_1) \\ & & +2(F_3\cdot F_4 & \cdots & +F_3\cdot F_4\cdots F_K & +F_3\cdot F_4\cdots F_1) \\ & & & \vdots & & \\ & & & & +2(F_3\cdot F_4\cdots F_K & +F_3\cdot F_4\cdots F_1) \\ & & & & & +2F_3\cdot F_4\cdots F_1 \\ -F_2 & -F_2\cdot F_3 & -F_2\cdot F_3\cdot F_4 & \cdots & -F_2\cdot F_3\cdot F_4\cdots F_K & -F_2\cdot F_3\cdot F_4\cdots F_1 \\ & -2(F_2\cdot F_3 & +F_2\cdot F_3\cdot F_4 & \cdots & +F_2\cdot F_3\cdot F_4\cdots F_K & +F_2\cdot F_3\cdot F_4\cdots F_1) \\ & & -2(F_2\cdot F_3\cdot F_4 & \cdots & +F_2\cdot F_3\cdot F_4\cdots F_K & +F_2\cdot F_3\cdot F_4\cdots F_1) \\ & & & \vdots & & \\ & & & & -2(F_2\cdot F_3\cdot F_4\cdots F_K & +F_2\cdot F_3\cdot F_4\cdots F_1) \\ & & & & & -2F_2\cdot F_3\cdot F_4\cdots F_1 \end{array}$$

The elements of the remaining columns of Σ_2 can be expressed in a similar manner. By summing these expressions, after cancelling out common terms, we obtain the following:

$$\Sigma_2 = -K^2C\Sigma_S + K(1-(2K-1)C) + 2\sum_{r=1}^{K}\sum_{i=1}^{K-1}\prod_{m=r}^{r+i-1} F_m$$

Hence, substituting into Eq. (12), we obtain the following:

$$\begin{aligned} E[n^2] &= \frac{K^2C}{1-C} + \frac{2K^2C}{(1-C)\Sigma_S} + \frac{-K^2C\Sigma_S + K(1-(2K-1)C) + 2\sum\limits_{r=1}^{K}\sum\limits_{i=1}^{K-1}\prod\limits_{m=r}^{r+i-1} F_m}{\Sigma_S(1-C)} \\ &= \frac{K(1+C) + 2\sum\limits_{r=1}^{K}\sum\limits_{i=1}^{K-1}\prod\limits_{m=r}^{r+i-1} F_m}{(1-C)\Sigma_S} \end{aligned}$$

■

B. Evaluation of S_j

1) CSD_{UA}: With uniform arbitration, when an allocated flow is unable to transmit, amongst those flows that currently perceive good channels, a flow is randomly selected for transmission. Considering flow 1's performance, flow 1 will transmit in a slot allocated to it as long as it perceives an error-free channel, i.e., $S_1 = p_G$, since it has the highest priority to transmit. In any slot that is not allocated to flow 1, due to our assumption of homogeneity, we have S_2=S_3=$\cdots$=S_K.

Let us consider a slot that is allocated for flow *j* and evaluate $S_j,\ 2 \le j \le K$. For flow 1 to transmit in this slot, the necessary conditions are that flow 1 must perceive an error-free channel and flow *j* must perceive an erroneous channel, and these occur with probability $p_G(1-p_G)$.

For the remaining K-2 flows, the probability that *m* flows will have error-free channels while the remaining K-2-*m* flows will have erroneous channels is given as follows:

$$\binom{K-2}{m} p_G^m (1-p_G)^{K-2-m}, \quad 0 \le m \le K-2$$

Suppose that *m*=M. In this case, there are M+1 flows contending for transmission, and therefore, the probability that flow 1 will be selected to transmit is $\frac{1}{M+1}$.

Putting all the terms together, we obtain the following expression:

$$\begin{aligned} S_j &= \sum_{m=0}^{K-2} \frac{\binom{K-2}{m} p_G^m (1-p_G)^{K-2-m} p_G(1-p_G)}{m+1} \\ & \quad 2 \le j \le K \end{aligned} \tag{13}$$

Multiplying the RHS of Eq. (13) by (K-1), we obtain the following:

$$\begin{aligned} & \sum_{m=0}^{K-2} \frac{(K-2)!(K-1)p_G^m(1-p_G)^{K-2-m}p_G(1-p_G)}{(K-2-m)!m!(m+1)} \\ =& \sum_{m=0}^{K-2} \frac{(K-1)!}{(K-2-m)!(m+1)!} p_G^m(1-p_G)^{K-2-m}p_G(1-p_G) \\ =& \sum_{m=0}^{K-2} \binom{K-1}{m+1} p_G^{m+1}(1-p_G)^{K-2-m}(1-p_G) \\ =& \sum_{w=1}^{K-1} \binom{K-1}{w} p_G^{w}(1-p_G)^{K-1-w}(1-p_G) \\ =& [\sum_{w=0}^{K-1} \binom{K-1}{w} p_G^{w}(1-p_G)^{K-1-w}](1-p_G) - (1-p_G)^K \end{aligned}$$

According to binomial theorem, $\sum\limits_{k=0}^{n} \binom{n}{k} x^k y^{n-k} = (x+y)^n$. Therefore, we have

$$\begin{aligned} (K-1)S_j &= (1-p_G) - (1-p_G)^K \\ S_j &= \frac{1-p_G-(1-p_G)^K}{K-1} \end{aligned}$$

Hence, for the homogeneous K-flow CSD_{UA} scheduler, when channel errors are uncorrelated, we have the following:

$$S_j = \begin{cases} p_G, & j=1; \\ \frac{1-p_G-(1-p_G)^K}{K-1}, & 2 \le j \le K. \end{cases} \tag{14}$$

2) $CSD_{PA}(P_h)$, K odd: In this case, $P_{max} = \frac{K-1}{2}$ and there are two candidate flows for each priority level, P>0. Recall that P_h specifies the lowest priority level for which a flow can contend for transmission in a given slot. Hence, if Pr_1 is the priority level of flow 1 in a slot allocated to flow *j*, then flow 1 cannot transmit if $Pr_1 > P_h$, i.e., S_j = 0.

Let us assume that $Pr_1 \leq P_h$. If Pr_1 = 1, i.e., *j*=$\frac{K+1}{2}$ or *j*=$\frac{3-k}{2}$, then we have the following:

$$\begin{aligned} S_{\frac{K+1}{2}} = S_{\frac{3-k}{2}} &= (1-p_G)p_G \sum_{m=0}^{1} \frac{\binom{1}{m} p_G^m (1-p_G)^{1-m}}{m+1} \\ &= \frac{(1-p_G)p_G(2-p_G)}{2} \end{aligned}$$

Similarly, if Pr_1 = 2, i.e., *j*= $\frac{K+3}{2}$ or *j* = $\frac{5-K}{2}$, then we have the following:

$$\begin{aligned} S_{\frac{K+3}{2}} = S_{\frac{5-K}{2}} &= (1-p_G)p_G[\sum_{m=0}^{1} \frac{\binom{1}{m} p_G^m (1-p_G)^{1-m}}{m+1}](1-p_G)^2 \\ &= \frac{(1-p_G)^3 p_G(2-p_G)}{2} \end{aligned}$$

In general, for $Pr_1 \leq P_h$, we have the following:

$$S_{\frac{K+3}{2}-Pr_1} = S_{\frac{K+1}{2}+Pr_1} = \frac{(1-p_G)^{2Pr_1-1} p_G(2-p_G)}{2}$$

Hence, for the homogeneous K-flow $CSD_{PA}(P_h)$ scheduler, when channel errors are uncorrelated and K is odd, we have the following:

$$S_j = \begin{cases} p_G, & j=1; \\ \frac{(1-p_G)^{2Pr_1-1} p_G(2-p_G)}{2}, & 1 \leq Pr_1 \leq P_h, j = \{\frac{K+3}{2} - Pr_1, \frac{K+1}{2} + Pr_1\}; \\ 0, & P_h + 1 \leq Pr_1 \leq P_{max}. \end{cases} \quad (15)$$

3) $CSD_{PA}(P_h)$, K even: In this case, $P_{max} = \frac{K}{2}$ and there are two candidate flows for each P>1, and one candidate flow each for P=0 and P=1. Consider the case where $j \neq 1$. If Pr_1 = 1, i.e., *j*=1+$\frac{K}{2}$, since it is the only candidate flow, we have the following:

$$S_{1+\frac{K}{2}} = (1-p_G)p_G$$

However, if Pr_1 = 2, i.e., *j*= $\frac{K}{2}$ or *j* = $\frac{K}{2}$+2, then we have the following:

$$\begin{aligned} S_{\frac{K}{2}} = S_{\frac{K}{2}+2} &= (1-p_G)^2 p_G \sum_{m=0}^{1} \frac{\binom{1}{m} p_G^m (1-p_G)^{1-m}}{m+1} \\ &= \frac{(1-p_G)^2 p_G(2-p_G)}{2} \end{aligned}$$

Next, if Pr_1 = 3, i.e., *j* = $\frac{K}{2}$-1 or *j* = $\frac{K}{2}$+3, then we have the following:

$$\begin{aligned} S_{\frac{K}{2}-1} = S_{\frac{K}{2}+3} &= (1-p_G)^2 p_G[\sum_{m=0}^{1} \frac{\binom{1}{m} p_G^m (1-p_G)^{1-m}}{m+1}](1-p_G)^2 \\ &= \frac{(1-p_G)^4 p_G(2-p_G)}{2} \end{aligned}$$

Hence, for the homogeneous K-flow $CSD_{PA}(P_h)$ scheduler, when channel errors are uncorrelated and K is even, we have the following:

$$S_j = \begin{cases} p_G, & j=1; \\ p_G(1-p_G), & j = \frac{K}{2}+1, Pr_1 = 1,\ P_1 \leq P_h; \\ \frac{(1-p_G)^{2Pr_1-2} p_G(2-p_G)}{2}, & j = \{\frac{K}{2} + 2 - Pr_1, \frac{K}{2} + Pr_1\}, 2 \leq Pr_1 \leq P_h; \\ 0, & P_h + 1 \leq Pr_1 \leq P_{max}. \end{cases} \quad (16)$$

VI. Application: Admissibility of Wireless Scheduler based on Efficiency Requirement and Determination of Receiver Buffer Size

We shall present some numerical results for the case of uncorrelated errors by considering an application of our analysis. Suppose we have a K-flow homogeneous scheduling scenario that is constrained by an efficiency requirement in terms of a minimum required overall throughput, η_{min}. We would like to determine whether a given scheduler can meet this requirement, and if so, to compute the required buffer length, N_Q, at the wireless receiver.

A. Admissibility of Wireless Scheduler based on η_{min}

The throughput of any flow is given as the average number of packet transmissions of that flow per slot and can be expressed in terms of E[n] as follows:

$$\text{flow throughput} = \frac{1}{E[n]}$$

For a homogeneous K-flow system, the overall throughput, η, is given as follows:

$$\eta = \frac{K}{E[n]}$$

Hence, in order to satisfy the overall throughput requirement, η_{min}, we must have the following:

$$\eta \geq \eta_{min}$$

The above equation can be expressed in terms of an upper bound on E[n] as follows:

$$E[n] \leq \frac{K}{\eta_{min}} \tag{17}$$

The expressions for E[n] for various schedulers are given as follows:

1) CSD_{UA}: From Eq. (14), we can evaluate Σ_S as follows:

$$\begin{aligned}
\Sigma_S &= p_G + (K-1)\frac{1 - p_G - (1-p_G)^{K-1}}{K} \\
&= p_G + 1 - p_G - (1-p_G)^K \\
&= 1 - (1-p_G)^K
\end{aligned}$$

Hence, we obtain the following:

$$E[n] = \frac{K}{\Sigma_S} = \frac{K}{1-(1-p_G)^K}$$

2) $CSD_{PA}(P_h)$: For odd values of K, using Eq. (15), we can evaluate Σ_S as follows:

$$\begin{aligned}
\Sigma_S &= p_G + 2\sum_{m=1}^{P_h} \frac{(1-p_G)^{2m-1}p_G(2-p_G)}{2} \\
&= p_G + \frac{(1-p_G)(2-p_G)p_G[1-(1-p_G)^{2P_h}]}{1-(1-p_G)^2} \\
&= p_G + \frac{(1-p_G)(2-p_G)p_G[1-(1-p_G)^{2P_h}]}{(1-(1-p_G)(1+(1-p_G))} \\
&= p_G + 1 - p_G - (1-p_G)^{2P_h+1} \\
&= 1 - (1-p_G)^{2P_h+1}
\end{aligned}$$

For even values of K, using Eq. (16), we have:

$$\begin{aligned}
\Sigma_S &= p_G + p_G(1-p_G) + 2\sum_{m=2}^{P_h} \frac{(1-p_G)^{2m-2}p_G(2-p_G)}{2} \\
&= 2p_G - p_G^2 + \frac{(1-p_G)^2(2-p_G)p_G[1-(1-p_G)^{2P_h-2}]}{1-(1-p_G)^2} \\
&= 2p_G - p_G^2 + \frac{(1-p_G)^2(2-p_G)p_G[1-(1-p_G)^{2P_h-2}]}{(1-(1-p_G)(1+(1-p_G))} \\
&= 2p_G - p_G^2 + 1 - 2p_G + p_G^2 - (1-p_G)^{2P_h} \\
&= 1 - (1-p_G)^{2P_h}, \; P_h \geq 1
\end{aligned}$$

Hence, we obtain the following:

$$E[n] = \begin{cases} \frac{K}{1-(1-p_G)^{2P_h+1}}, & \text{K odd;} \\ \frac{K}{1-(1-p_G)^{2P_h}}, & \text{K even.} \end{cases} \tag{18}$$

3) $CSD_{PA}(P_h = P_{max})$: Substituting for $P_h = P_{max}$ in Eq. (18), we obtain E[n] = $\frac{K}{1-(1-p_G)^K}$ which is the same as that obtained for the CSD_{UA} scheduler.

4) $CSD_{PA}(P_h = 0)$: This corresponds to the simple WRR scheduler, since in this case, flows are only allowed to transmit in slots allocated to them. Substituting for P_h = 0 into Eq. (18), we obtain E[n] = $\frac{K}{p_G}$, which is the *worst-case* expected HOL packet delay of CSD schedulers.

The expression for E[n^2] is given as follows:

$$E[n^2] \quad = \quad \frac{K^2(2-p_G)}{p_G^2}$$

5) Fair Aggregation (FA) Scheduler: We consider the FA scheduler [19] as a variant of the WRR scheduler, where the motivation is to reduce the delay variance relative to the WRR scheduler while retaining the channel efficiency. Instead of switching resource allocation between flows at each slot, this is performed upon each *successful* packet transmission. Hence, the FA flow transmission sequence corresponds to the WRR flow allocation sequence and is shown in Fig. 3. In this way, the likelihood that a flow will transmit at least once within each cycle of K slots is increased compared to the WRR scheduler.

According to Fig. 3, after each flow 1 transmission, the flow transmission sequence before the next flow 1 transmission is fixed and given by $\{2, 3, 4, \cdots K\}$. These K-1 packets must be transmitted in the first *n*-1 slots before flow 1 transmits in slot *n*. Hence, in the first *n*-1 slots, there must be exactly K-1 slots where the channel is error-free; in addition, the channel must be error-free in slot *n*. Therefore, we can write the pdf of *n* as follows:

$$d(n) = \begin{cases} \binom{K+i-1}{i}(1-p_G)^i p_G^K, & n = K+i,\ i \geq 0; \\ 0, & n < K. \end{cases} \tag{19}$$

From Eq. (19), we can compute E[n] as follows:

$$\begin{aligned} E[n] &= \sum_n n \cdot d(n) \\ &= \sum_{i=0}^{\infty} (K+i) \binom{K+i-1}{i} (1-p_G)^i p_G^K \\ &= \sum_{i=0}^{\infty} (K+i) \frac{(K+i-1)!}{i!(K-1)!} (1-p_G)^i p_G^K \\ &= \sum_{i=0}^{\infty} \frac{K(K+i)!}{K \cdot i!(K-1)!} (1-p_G)^i p_G^K \\ &= K \cdot p_G^K \sum_{i=0}^{\infty} \binom{K+i}{i} (1-p_G)^i \end{aligned}$$

Using the following result from Binomial theorem:

$$\sum_{i=0}^{\infty} \binom{K+i}{i} x^i \quad = \quad \frac{1}{(1-x)^{K+1}} \tag{20}$$

we obtain

$$E[n] \quad = \quad \frac{K \cdot p_G^K}{(1-(1-p_G))^{K+1}} = \frac{K}{p_G}$$

Similarly, we can compute E[n^2] as follows:

$$\begin{aligned} E[n^2] &= \sum_n n^2 \cdot d(n) \\ &= \sum_{i=0}^{\infty} (K+i)^2 \binom{K+i-1}{i} (1-p_G)^i p_G^K \\ &= K. \sum_{i=0}^{\infty} (K+i) \binom{K+i}{i} (1-p_G)^i p_G^K \end{aligned}$$

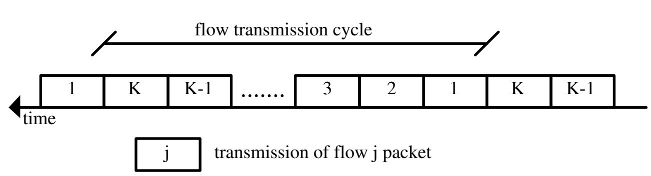
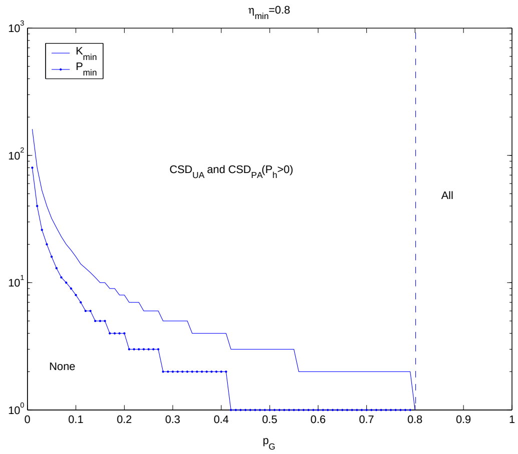
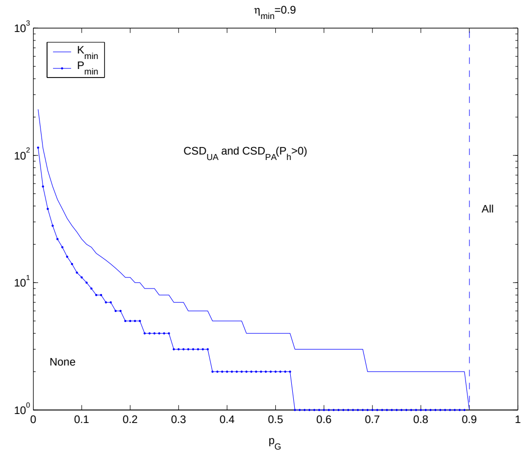


Fig. 3. Flow transmission sequence in K-flow homogeneous FA scheduling



(a)



(b)

Fig. 4. Operating Regions (K,P_h,p_G) for various schedulers to satisfy η_{min} = (a) 0.8 and (b) 0.9

Differentiating Eq. (20), we obtain the following:

$$\sum_{i=0}^{\infty} i \binom{K+i}{i} x^{i-1} = \frac{K+1}{(1-x)^{K+2}}$$

Hence, we obtain

$$\begin{aligned} E[n^2] &= K^2 \sum_{i=0}^{\infty} \binom{K+i}{i} (1-p_G)^i p_G^K \\ &+ K \sum_{i=0}^{\infty} i \binom{K+i}{i} (1-p_G)^i p_G^K \\ &= K^2 \frac{p_G^K}{p_G^{K+1}} + K \cdot p_G^K (1-p_G) \frac{K+1}{p_G^{K+2}} \\ &= \frac{K^2 + K - K \cdot p_G}{p_G^2} \end{aligned}$$

We substitute the expressions of E[n] for each wireless scheduler into Eq. (17) and illustrate the constraint graphically in Fig. 4 with η_{min}= 0.8 and 0.90 respectively.

We can partition the operating region into three sub-regions. The region denoted by *All*, given by $p_G \geq \eta_{min}$, indicates that all schedulers can be deployed for any K while satisfying the throughput constraint. On the other extreme, the region denoted by *None*, where $K < K_{min}$ for CSD_{UA} and $P_h < P_{min}$ for $CSD_{PA}(P_h)$, indicates that none of the schedulers can satisfy the throughput constraint. The remaining region stipulates the requirements on K and P_h for CSD schedulers to satisfy the requirement. Hence, given η_{min}, K and p_G, we can determine which of the scheduler(s) are admissible with respect to η_{min}.

B. Determination of N_Q in terms of HOL Packet Delay Statistics

A G/G/1 system can be used to model the queueing behavior at a wireless receiver, as shown in Fig. 5. Based on queueing theory, the average waiting time in the queue under steady-state conditions satisfies the following [20]:

$$W \leq \frac{\lambda(\sigma_a^2 + \sigma_b^2)}{2(1-\rho)} - \frac{\lambda(1-\rho)\sigma_a^2}{2} \tag{21}$$

where

$$\begin{aligned} \sigma_a^2 &= \text{Variance of inter-arrival times} \\ \sigma_b^2 &= \text{Variance of the service times} \\ \lambda &= \text{Average arrival rate} \\ \frac{1}{\mu} &= \text{Average service time} \\ \rho &= \text{Utilization factor} = \frac{\lambda}{\mu} \end{aligned}$$

If we assume a constant-rate server (i.e., σ_b=0), and a constant utilization factor ρ, then Eq. (21) can be written as follows:

$$W \leq C\lambda\sigma_a^2, \; where \; C = \frac{\rho(2-\rho)}{2(1-\rho)} = constant$$

Using Little's formula, which is valid when steady-state conditions exists, the average number of packets waiting in the queue can be expressed as follows:

$$\begin{aligned} N_W &= \lambda W \\ &\leq C\lambda^2\sigma_a^2 \end{aligned}$$

Hence, we can choose a suitable N_Q in terms of arrival statistics, λ and σ_a, as follows:

$$N_Q = C\lambda^2\sigma_a^2$$

However, we note the following relationship between arrival statistics to the buffer and departure statistics from the scheduler:

$$\begin{aligned} \lambda &= \frac{1}{E[n]} \\ \sigma_a^2 &= Var[n] \end{aligned}$$

where

$$Var[n] = E[n^2] - (E[n])^2$$

Hence, N_Q can be computed in terms of the HOL packet delay statistics of the wireless scheduler as follows:

$$N_Q = C \cdot \frac{Var[n]}{E[n]^2}$$

From Section VI-A, we can obtain closed-form expressions for the ratio $\frac{Var[n]}{E[n]^2}$ for the WRR and FA schedulers as follows:

$$\frac{Var[n]}{E[n]^2} = \begin{cases} 1-p_G, & \text{WRR scheduler;} \\ \frac{1-p_G}{K}, & \text{FA scheduler.} \end{cases} \quad (22)$$

In addition, we have the following:

Theorem 4: For uncorrelated channel errors, the ratio $\frac{Var[n]}{E[n]^2}$ for a homogeneous K-flow CSD scheduler is asymptotically upper bounded.

Proof: From Theorem 2 and 3, we have the following expression:

$$\frac{Var[n]}{E[n]^2} = \frac{K(1+C) + 2\sum_{r=1}^{K}\sum_{i=1}^{K-1}\prod_{m=r}^{r+i-1} F_m}{(1-C)\Sigma_S} \cdot \frac{\Sigma_S^2}{K^2}$$

Since $F_j \leq 1$, 1$\leq j \leq$ K, we have the following:

$$\begin{aligned} \frac{Var[n]}{E[n]^2} &\leq \frac{K(1+C) + 2\sum_{r=1}^{K}\sum_{i=1}^{K-1}}{(1-C)\Sigma_S} \cdot \frac{\Sigma_S^2}{K^2} \\ &= \frac{K(1+C) + 2K(K-1)}{(1-C)\Sigma_S} \cdot \frac{\Sigma_S^2}{K^2} \\ &= \frac{K(2K-1+C)}{(1-C)\Sigma_S} \times \frac{\Sigma_S^2}{K^2} \\ &= \frac{2\Sigma_S}{1-C} - \frac{\Sigma_S}{K} \end{aligned}$$

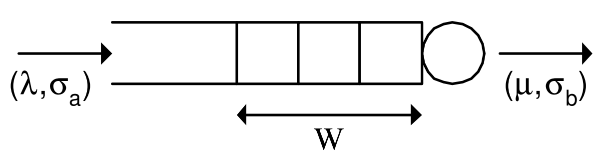
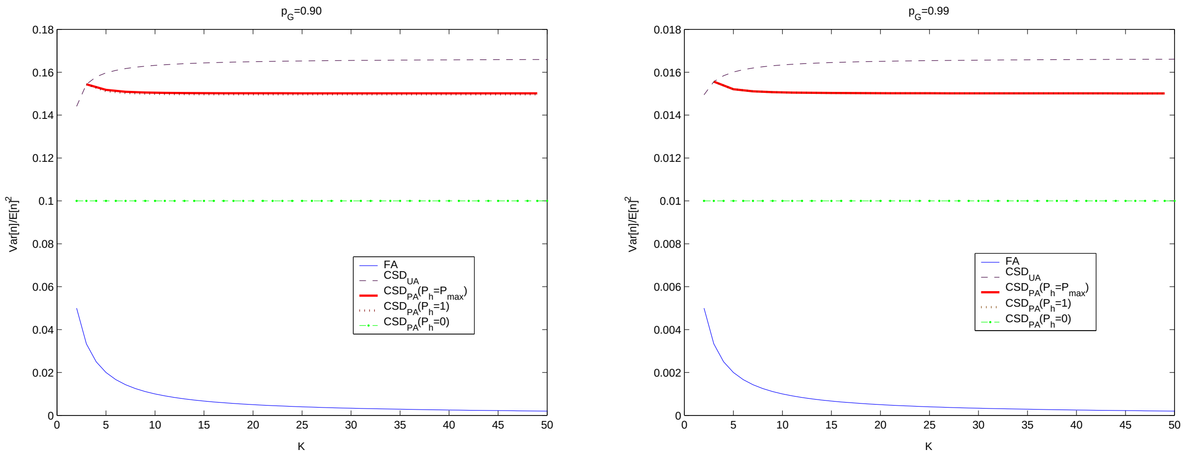


Fig. 5. G/G/1 Queueing representation of receiver buffer



(a) (b)

Fig. 6. $\frac{Var[n]}{E[n]^2}$ of various schedulers for p_G = (a) 0.9 and (b) 0.99

From Section VI-A, we know that 0$\leq \Sigma_S \leq 1$. Hence,

$$\frac{Var[n]}{E[n]^2} \leq \frac{2\Sigma_S}{1-C}$$

Since 0$\leq C \leq 1$, the RHS of the above equation is finite and hence, the ratio $\frac{Var[n]}{E[n]^2}$ is upper bounded. ■
Hence, the queueing system at the buffer is *stable* for the wireless schedulers considered in our analysis.

For a throughput constraint of η_{min} = 0.90, we would like to determine the admissibility and compare the ratio $\frac{Var[n]}{E[n]^2}$ for each scheduler for two cases: (1) $p_G \geq \eta_{min}$ and (2) $p_G < \eta_{min}$.

1) $p_G \geq \eta_{min}$: According to Fig. 4, in this region, all schedulers are admissible in terms of the throughput constraint. We plot the ratio $\frac{Var[n]}{E[n]^2}$ for each scheduler as a function of K for p_G = 0.9 and 0.99 in Fig. 6.

We can categorize the FA and the $CSD_{PA}(P_h = 0)$ schedulers as channel-unaware schedulers since their transmission heuristics are independent of the channel process. Since the channel conditions are very good, most flows transmit in their allocated slots. Hence, the additional transmission opportunities available for channel-aware schedulers actually results in an increased buffer requirement since delay variance is increased while the mean delay is reduced compared to channel-unaware schedulers. However, the lower buffer requirement of channel-unaware schedulers is traded-off with lower channel efficiency compared with channel-aware schedulers.

Under very good channel conditions, the expression of E[n] for the CSD_{UA} and $CSD_{PA}(P_h{=}P_{max})$ can be approximated by K. As K increases, the *randomness* of transmissions (i.e., Var[n]) in the CSD_{UA} scheduler increases while the $CSD_{PA}(P_h = P_{max})$ scheduler becomes more effective in controlling the *randomness* of transmission. As a result, although E[n] increases with K, the buffer requirements of the CSD_{UA} scheduler increases while that of the $CSD_{PA}(P_h = P_{max})$ scheduler decreases as K increases since Var[n] is the dominant term in the ratio. However, for large K, the E[n] term becomes dominant and hence, the ratio converges asymptotically for both schedulers. The buffer size requirement for $P_h < P_{max}$ is similar to that of $P_h = P_{max}$ since most transmissions take place in allocated slots under very good channel conditions.

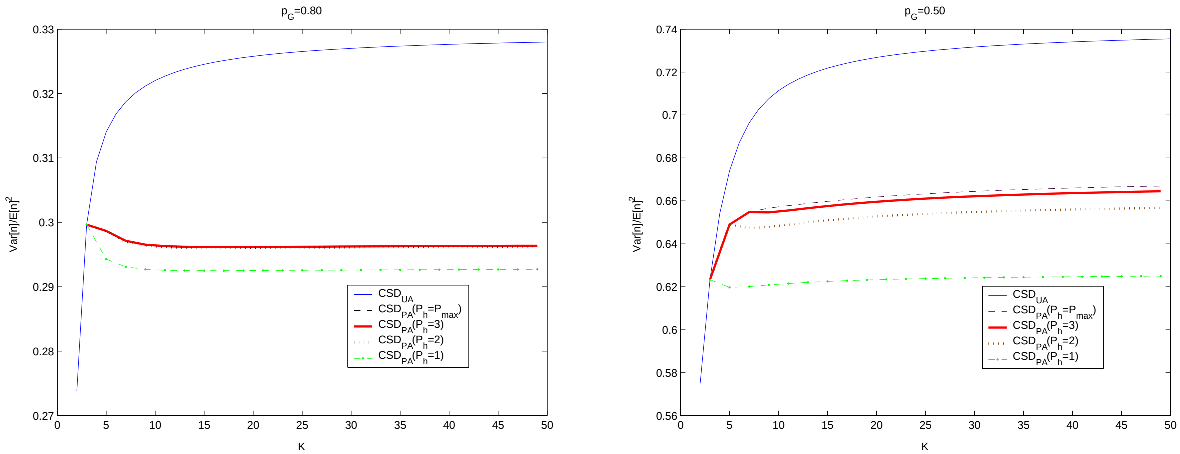
2) $p_G < \eta_{min}$: According to Fig. 4, in this region, the FA and $CSD_{PA}(P_h = 0)$ schedulers are inadmissible. Hence, we compare $\frac{Var[n]}{E[n]^2}$ of the CSD_{UA} and $CSD_{PA}(P_h > 0)$ schedulers as a function of K for p_G = 0.8 and 0.5 in Fig. 7.

We observe the same trend between CSD_{UA} and $CSD_{PA}(P_h = P_{max})$ schedulers as when the channel conditions are very good. However, the buffer size requirement for the $CSD_{PA}(P_h)$ scheduler is reduced as P_h is reduced. Under poor channel conditions, a substantial amount of transmissions occur in non-allocated slots, and therefore, a smaller P_h is effective in limiting these transmissions, thereby reducing the delay variance, at the expense of reduced channel efficiency.

C. Effects of non-zero channel error correlation

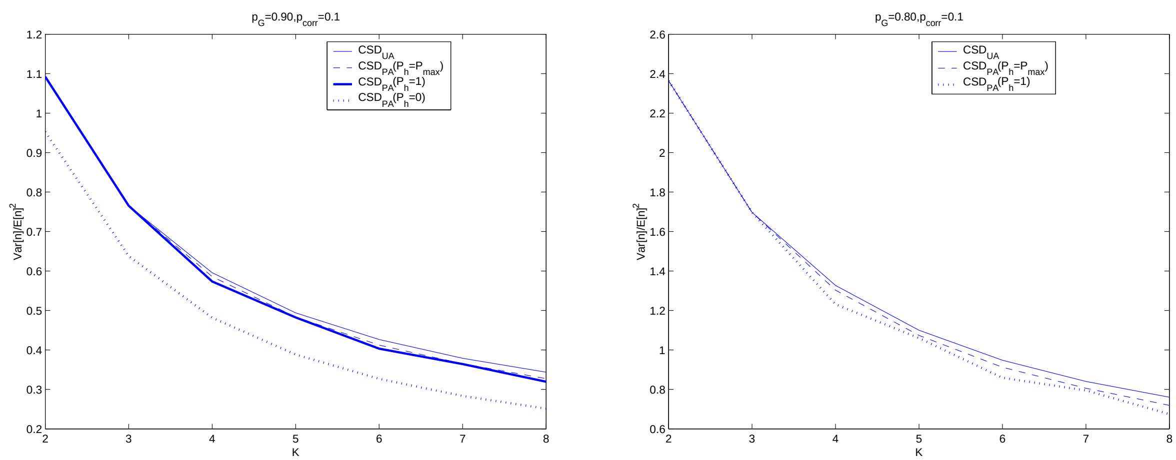
In this section, for a throughput constraint of η_{min} = 0.90, we would like to determine the admissibility and compare the ratio $\frac{Var[n]}{E[n]^2}$ for each scheduler for two cases: (1) $p_G \geq \eta_{min}$ and (2) $p_G < \eta_{min}$, when p_{corr} = 0.1.

1) $p_G \geq \eta_{min}$: According to Fig. 4, in this region, all schedulers are admissible in terms of the throughput constraint when channel errors are uncorrelated. In [19], we observed that with respect to the case of uncorrelated errors, the throughput achieved by the FA scheduler is degraded significantly as the level of error correlation is increased, while that achieved with the WRR and CSD schedulers is invariant with the level of error correlation. Hence, the FA scheduler is not admissible when channel errors are correlated.



(a) (b)

Fig. 7. $\frac{Var[n]}{E[n]^2}$ of various schedulers for p_G = (a) 0.8 and (b) 0.5



(a) (b)

Fig. 8. $\frac{Var[n]}{E[n]^2}$ of various schedulers for p_G = (a) 0.90 and (b) 0.80

We plot the ratio $\frac{Var[n]}{E[n]^2}$ for the WRR and CSD schedulers as a function of K for p_G = 0.90 in Fig. 8(a). As K increases, if a flow misses its allocated slot due to channel errors, it is less likely to miss the next allocated slot since the duration till the next allocation is increased. Therefore, the delay variation (i.e., Var[n]) due to transmission in non-allocated slots is reduced and therefore, the ratio $\frac{Var[n]}{E[n]^2}$ is reduced. We also note that the ratio tends asymptotically to the corresponding value achieved for uncorrelated errors. Therefore, the buffer system at the wireless receiver is stable. In addition, the relative buffer requirements amongst the schedulers are preserved as in the case of uncorrelated errors.

2) $p_G < \eta_{min}$: According to Fig. 4, in this region, the WRR scheduler is inadmissible. Hence, we compare the ratio $\frac{Var[n]}{E[n]^2}$ for different arbitration schemes of the CSD scheduler as a function of K for p_G = 0.8 in Fig. 8(b). Similar trends are observed in terms of the relative buffer requirements amongst the schedulers as for $p_G = 0.90$.

VII. Conclusions

In this paper, we developed a framework for the stochastic analysis of the delay performance of channel-state dependent wireless schedulers. These schedulers differ in the mechanism of choosing the 'instantaneous' best flow (arbitration scheme) to transmit based on available channel information in order to satisfy some performance requirements. We adopted a generic scheduling architecture based on proposed wireless schedulers in the literature, and defined variants that differ in terms of the arbitration scheme. We derived the delay probability density function and its moments for a Two-State Markov Chain Error Model using a matrix formulation approach.

We demonstrated the applicability of our analysis numerically by considering the admissibility of a wireless scheduler in terms of a minimum throughput requirement. This translates to an upper bound on the mean HOL packet delay. Given the channel condition and the total number of flows, we illustrated graphically the admissible region of each scheduler. Subsequently, we evaluated the buffer size requirement of the wireless receiver and highlighted the trade-off between buffer size requirements and channel efficiency.

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