Ad Hoc Networks with Topology-Transparent Scheduling Schemes: Scaling Laws and Capacity/Delay Tradeoffs

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***Abstract*— In this paper we investigate the limiting properties, in terms of capacity and delay, of an ad hoc network employing a topology-transparent scheduling scheme. In particular, we focus on Time-Spread Multiple Access (TSMA) protocols, which are able to offer, in a distributed fashion, a deterministic upper bound on the access delay. The analysis is based on some asymptotic (focusing) properties of geometric random graphs. The analytical framework is applied to both static and mobile networks. The obtained results are compared with the results present in the literature for the case of an optimum (centralized) scheduling scheme.**

I. Introduction

Starting from the seminal work of Gupta and Kumar on the capacity of wireless networks [1], various researchers have, in the last few years, addressed the limiting performance, in terms of throughput and delay, of ad hoc networks. Among the milestones in the area, we recall the work of Grossglauser and Tse [2], who proved that, by exploiting the multiuser diversity provided by node mobility, capacity can be increased with respect to the case of static nodes, at the expense of increased packet delay. Subsequent works have deeply investigated the arising capacity/delay tradeoffs [3], [4], [5], [6], [7].

In [1] the per-connection throughput in a random network consisting of n nodes is shown to scale as $\lambda(n) = \Theta\left(\sqrt{\frac{1}{n \log n}}\right)$.[1] Such scaling comes from the necessity of keeping the transmission range as small as possible, in order to limit the level of interference in the network. On the other hand, a scaling for the transmission range of the order $R(n) = \Theta\left(\sqrt{\frac{\log n + c(n)}{n}}\right)$, with $c(n) \to +\infty$ is necessary and sufficient to ensure connectivity of the resulting network [8], [9]. Hence, in dense wireless networks, interference is the factor affecting the (negative) scaling properties of the throughput. Furthermore, all results rely on the assumption of a centralized perfect schedule of transmissions, representing thus upper bounds on the performance achievable by a distributed scheduling scheme.

In this paper, we present some results, in terms of scaling laws and capacity/delay tradeoffs, for ad hoc networks employing Time-Spread Multiple Access (TSMA) protocols. TSMA schemes (also referred to as *topology-transparent scheduling methods*) were introduced in the 90s following the pioneering work of Chlamtac and Faragó [10]. Their approach exploits some properties of Galois fields to design a fully distributed scheduling algorithm, based on a frame structure. The frame is divided into q^2 slots, and each node attempts transmissions q times per frame. Even if the overall protocol is not collision-free, each node is ensured to successfully transmit at least once per frame. The first remarkable property of these protocols is that they are able to provide a deterministic guaranteed upper-bound (equal to the frame length in the static case, since the success pattern is deterministic, and to twice the frame length in the mobile case, where neighbors may change from frame to frame) on the access delay at each node. TSMA and similar schemes are referred also to as topology-transparent scheduling mechanisms, in that they do not require each node to have detailed topological information. The schedules depend indeed only on two design parameters, namely the number of nodes in the network, n, and the maximum degree of the network, Δ, defined as the maximum number of interferers of any node in the network. In this paper, we exploit some focusing properties of Δ in geometric random graphs [11] for studying the asymptotic behavior of such protocols in a highly dense ad hoc network.

The contribution of the paper lies in two properties of TSMA protocols, which put the results presented aside from the ones in the literature:

- in all works presented in the literature a centralized scheduling (TDM-like) scheme for transmissions is assumed, while we consider a fully distributed scheme. While the results presented for centralized TDM schemes could, in principle, be extended to distributed implemen-

[1] Throughout the paper, we will use the following notation. Given two functions $f(n)$ and $g(n)$, we say that: (i) $f(n) = o(g(n))$ if $\frac{f(n)}{g(n)} \to 0$ as $n \to +\infty$, (ii) $f(n) = O(g(n))$ if $\frac{f(n)}{g(n)}$ is upperbounded for n large enough, (iii) $f(n) = \omega(g(n))$ if $g(n) = o(f(n))$, (iv) $f(n) = \Omega(g(n))$ if $g(n) = O(f(n))$ and (v) $f(n) = \Theta(g(n))$ if $f(n) = O(g(n))$ and $f(n) = \Omega(g(n))$.

tations, this would require the exchange of control and signaling messages for maintaining detailed knowledge of the network topology (with a remarkable traffic burden in the case of mobile devices);

- TSMA schemes offer a deterministically upper-bounded delay for packet transmission, which leads to scaling laws in terms of *worst-case (maximum)* access delay rather than *average* delay. This can be exploited to engineer solutions able to offer Quality-of-Service guarantees.

The remainder of the paper is organized as follows. In Sec. II, we introduce the system model and recall some useful results from Geometric Random Graphs theory. In Sec. III, scaling laws are derived for the network capacity, packet delay and energy efficiency for the cases of both static and mobile nodes. In Sec. IV some extensions of the proposed model are proposed, along with some numerical results. Sec. V concludes the paper pointing out some promising directions for future research.

II. System Model

A. Topology-Transparent Scheduling Schemes

A fundamental requirement in radio transmission is to avoid/limit the interference coming from neighboring devices, controlling the access to the shared medium. Classically, two main categories of access control schemes have been considered, namely scheduled mechanisms and random ones. In the first case, transmissions are scheduled (by a central entity, in principle), so that collision-free access may be granted to the nodes. Examples of such class include standard TDMA mechanisms. In the second case (comprising, e.g., the classical ALOHA scheme and all CSMA variations), nodes attempt independently to access the channel, resulting in potential collisions and demanding the implementation of a collision resolution protocol [12]. Scheduled channel access schemes present considerable advantages over random access, in terms of (i) better channel utilization (ii) better scalability properties in the presence of a large number of heavily loaded nodes (iii) the possibility of offering quality-of-service guarantees and traffic differentiation. Scheduled mechanisms can also be implemented in a distributed way, by means of the exchange of appropriate signaling messages, so that every node has a complete knowledge of the network topology. When nodes are moving, however, the burden due to signalling traffic increases and may end up using most of the available bandwidth, causing starvation problems to data flows. Topology-transparent schemes were introduced by Chlamtac and Faragó in 1994 to overcome these problems [10]. Their operations are based on some properties of finite (Galois) fields. Time is divided into frames, each frame being subdivided in q subframes, where q is the power of a prime number. Each subframe is subdivided into q time slots, so that each frame comprises q^2 slots. Each node transmits once per subframe. Not all transmissions are granted collision-free, but it can be ensured (by an appropriate choice of the design parameter q), that all nodes are able to transmit successfully at least once per frame. The frame structure is depicted in Fig. 1 for the case $q = 3$. TSMA schemes involve the assignment to each node of a unique

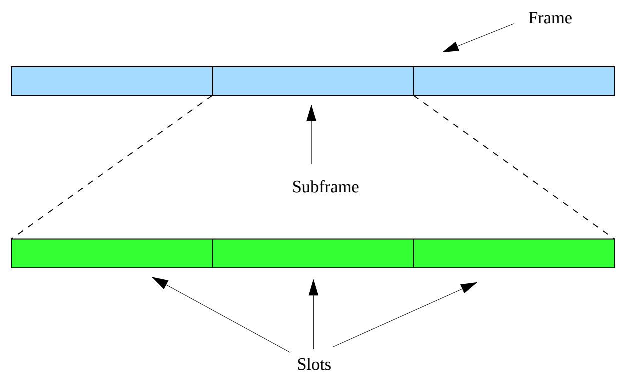

Fig. 1. Sketch of a topology-transparent scheme frame structure with $q = 3$.

polynomial of degree k over the finite field $GF(q)$, which determines the transmissions schedule. Overlap of slot sets corresponds to common roots of the polynomial associated to different nodes. In order to perform successful scheduling of packet transmissions, the mechanism relies on an appropriate choice of q and k, to ensure that a slot cannot be covered by up to Δ^2 nodes, where Δ is the maximum node degree. The non-covering condition can be expressed as [10]

$$q \geq k\Delta + 1. \tag{1}$$

On the other hand, polynomials must be uniquely assigned, so that the following condition must hold as well:

$$q^{k+1} \geq n, \tag{2}$$

where n is the total number of nodes in the network.

Topology-transparent schemes have not received considerable attention in the literature: a variant of the original scheme, able to maximize the network throughput, has been presented in [13]. A (recursive) generalization of the scheme in [10] has been presented in [14]. Recently, Syrotiuk et al. have shown that all the schemes in this family can be represented by orthogonal arrays [15]. Lately, it has also been shown in [16] that the throughput of topology-transparent scheduling schemes can be enhanced by transmitting (with a given probability) in slots that are not assigned for transmission.

B. Ad Hoc Networks and Random Graphs

We assume that n nodes are distributed on a unit-area torus[2] in an i.i.d. fashion according to a general density f, assumed to be bounded and non-zero, i.e., $\forall a$, $0 < f_{min} \leq f(a) \leq f_{max} < +\infty$. All nodes transmit at a fixed power P_{tx}; the case of power controlled devices should be analyzed in terms of achievable rate regions [18] or total network capacity [19] and is left for future work. We consider an additive white Gaussian noise channel, with a path loss of the form $\frac{1}{d^\alpha}$, $\alpha \geq 2$, and denote by W the noise power. In our model, the packet error rate versus signal-to-noise ratio is approximated by a step-like function, the threshold being denoted by Ψ. Such

[2]The torus assumption is taken in order to avoid problems related to edge effects; nonetheless, results can be generalized to a unit square. Also, the analysis can be applied to the case of nodes distributed on the infinite plane, scaling the area, and not the density, with the number of nodes n, as in [17].

a model works, e.g., in the case when good long codes are used. It corresponds to the situation in which there exists a fixed value R (called the communication range) such that a node can successfully decode a message if and only if its distance from the source is less than or equal to R. Under such assumptions, the network can be modeled as a Geometric Random Graph [11], [20]. In Fig. 2, we reported the network coverage (intended as the union of the area covered by each circle) and the resulting connectivity graph for the case of $n = 1000$ nodes uniformly scattered over a 100×100 m^2 areas and having a communication range $R = 3.73$ m.

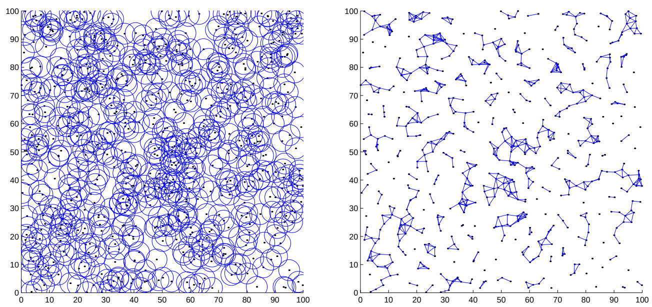

Fig. 2. An ad hoc network with deterministic transmission range and its equivalent connectivity graph.

The communication range R takes the form: $R = \left(\frac{KP_{tx}}{W\Psi}\right)^{\frac{1}{\alpha}}$, where K accounts for the antenna gains. In the paper, we aim at investigating the limiting properties of such a network model by appropriately scaling P_{tx} as a function of n, and therefore write $P_{tx}(n)$. Accordingly, the communication range, as a function of n, will be denoted by $R(n)$. The neighbors of a given node are defined as the nodes with which it is able to communicate, i.e., the nodes within distance $R(n)$. As above, we define the maximum degree of the network $\Delta(n)$ as the maximum number of *interfering* nodes, i.e., nodes within the interference range $\hat{R}(n)$. In radio transmissions with current technologies, the interference range is known to be larger than the communication range. Nonetheless, since we focus on the asymptotic behavior of the network, and are interested in obtaining scaling laws, we can safely assume that the two quantities are linked by a (finite) multiplicative factor [3], so that they follow the same limiting behavior. Hence, under such assumption $\Delta(n)$ scales as the maximum number of neighbors, i.e., the maximum number of nodes within distance $R(n)$.

In order to assess the asymptotic performance of the network, we can apply some results on the limiting properties of Geometric Random Graphs [11]. In particular, we recall the following results:

Proposition 1: The following holds:

(i) if there exist $k \in \mathbb{N}$ such that $n^{\frac{k+1}{k}} R^2(n) \to +\infty$ and $n^{\frac{k+2}{k+1}} R^2(n) \to 0$, then:

$$\mathbb{P}[\Delta(n) = k] \to 1 \text{ as } n \to +\infty; \tag{3}$$

(ii) if $\frac{nR^2(n)}{\log n} \to 0$, $nR^2(n) \to +\infty$ and $\frac{\log \frac{1}{nR^2(n)}}{\log n} \to 0$, then:

$$\lim_{n\to+\infty} \left[\frac{\Delta(n) \log\left(\frac{\log n}{nR^2(n)}\right)}{\log n} \right] = 1 \tag{4}$$

in probability;

(iii) if $R(n) \to 0$ and $\frac{nR^2(n)}{\log n} \to \gamma$ for $0 < \gamma \le +\infty$, then:

$$\lim_{n\to+\infty} \left(\frac{\Delta(n)}{nR^2(n)}\right) = \beta \tag{5}$$

$\mathbb{P}$-almost surely[4], where β and γ are defined in [11, Thm. 6.14];

(iv) if $R(n) = \Theta(1)$, then:

$$\Delta(n) = \Theta(n). \tag{6}$$

In order to better understand, from a networking point of view, the four situations considered in the proposition above, we consider the node isolation probability. We recall that the node isolation probability, defined as the probability that any given node is not able to communicate with any of its neighbors, can be well approximated assuming a Poisson distribution for the number of nodes [11], getting $P_I \approx e^{-n\pi R^2(n)} = n^{-\pi \frac{nR^2(n)}{\log n}}$ [21]. We hence note that cases (i) and (ii) correspond to the situation in which all the nodes are isolated $\mathbb{P}$-almost surely. This situation is clearly of little interest from the point of view of TSMA protocols (at least in the static case, see below), since it implies that interference and contentions are not an issue in the network. On the other hand, case (iii) corresponds to the situation in which the nodes are isolated with probability $P_I \to n^{-\pi\gamma} \to 0$, i.e., no isolated nodes are $\mathbb{P}$-a.s. present. This case covers all the situations in which the nodes are starting to get connected, with no isolated nodes but no assurance of the network being connected. Further, note that the value of β is upper-bounded by f_{max}, the maximum taken by the density function f [11]. Finally, case (iv) corresponds to the situation in which the network reduces to a single-hop domain, and no store-and-forward operations are necessary. In the following section, we thus concentrate our attention on cases (iii) and (iv) in the Proposition above.

III. Scaling Laws and Capacity/Delay Tradeoffs

A. The Static Case

Let us first consider the case in which all nodes are static. The performance of TSMA schemes heavily depends on the maximum node degree. This parameter is indeed needed to design a scheme that is able to ensure that, within a frame, each node will be able to perform at least one successful transmission. The original Chlamtac-Faragó scheme achieves a frame length satisfying [10]:

$$L(n) = O\left(\frac{\Delta^2(n)\log^2 n}{\log^2 \Delta(n)}\right). \tag{7}$$

Note that the scheme of [10] pursues the minimum frame length as design goal; however its performance in terms of

[3] In reality, the slightly milder condition $R(n) = \Theta(\hat{R}(n))$ suffices.

[4] Convergence $\mathbb{P}$-almost surely (abbreviated as $\mathbb{P}$-a.s. in the remainder of the paper) means that it holds everywhere except for a subset of Ω with measure induced by $\mathbb{P}$ equal to 0.

throughput is not optimal. An alternative but similar scheme, which aims at maximizing the minimum throughput, is reported in [13].

On the other hand, we also need a lower bound on the frame length. In general, a bound of the order $\Omega(\log n)$ is known [22], which does not depend on Δ. In our case, since we will control the scaling for Δ, a bound of the type:

$$L(n) = \Omega\left(\Delta^2(n)\right), \tag{8}$$

will be used, which can be easily derived from (1) as $q \geq \Delta$ and $L = q^2$.

Since the frame length corresponds to the maximum access delay that each node will incur, the *delay* performance of TSMA depends just on the parameter Δ. While in principle, for any random distribution of nodes, this cannot *a priori* be ensured to be bounded (and its estimation would require the exchange of control signals with a consequent traffic burden in the case of highly mobile devices), we can nonetheless exploit the properties of geometric random graphs presented in the previous section to derive scaling laws for both throughput and packet delay.

Let us consider a scaling of the form $P_{tx}(n) = \frac{\Psi W}{K}\left(\frac{\log n + c(n)}{n}\right)^{\frac{\alpha}{2}}$ for the transmission power. This leads to a scaling of the form $R(n) = \sqrt{\frac{\log n + c(n)}{n}}$ for the communication range. We recall from [8], [9] that, with a scaling of this form, $c(n) \to +\infty$ is necessary and sufficient to ensure connectivity of the resulting network with unitary probability.

With respect to case (iii) in Proposition 1, we note that it corresponds to the situation in which $c(n) = o(n)$, and we have:

$$\gamma = 1 + \lim_{n \to +\infty} \frac{c(n)}{\log n}. \tag{9}$$

Hence, the parameter γ is finite if and only if $c(n) = O(\log n)$, while it tends to infinity in the case $c(n) = \omega(\log n)$. Using then the results of Proposition 1 together with (7), we obtain the following results:

Proposition 2: Given $P_{tx}(n) = \frac{\Psi W}{K}\left(\frac{\log n + c(n)}{n}\right)^{\frac{\alpha}{2}}$, the following holds:

(i) if $c(n) \to c$, $0 \leq c \leq +\infty$, then:

$$L(n) = O\left(\frac{[\log n + c(n)]^2 \log^2 n}{\log^2 [\log n + c(n)]}\right); \tag{10}$$

$$L(n) = \Omega\left([\log n + c(n)]^2\right). \tag{11}$$

If $c(n) = \Theta(\log n)$ (which implies $\gamma < +\infty$ in Proposition 1), the equation above simplifies to:

$$L(n) = O\left(\frac{\log^4 n}{\log^2(\log n)}\right); \tag{12}$$

$$L(n) = \Omega\left(\log^2 n\right). \tag{13}$$

(ii) if $c(n) = \Theta(n)$, then:

$$L(n) = \Theta(n^2). \tag{14}$$

Proof: The results follow directly using (7) and (8) together with the results in Proposition 1. ∎

The proposition above defines the network performance in terms of frame length and hence *maximum access delay*. The network capacity can be lower-bounded considering that in any frame, each node is able to transmit successfully at least one message, getting:

$$S(n) = \Omega\left(\frac{n}{L(n)}\right) = \Omega\left(\frac{n \log^2 \Delta(n)}{\Delta^2(n) \log^2 n}\right). \tag{15}$$

On the other hand, we can upper-bound the network capacity by considering that all nodes are always successful when attempting transmissions on the radio channel. From the protocol definition, the number of successful attempts per frame is $T(n) = \sqrt{L(n)}$, getting:

$$S(n) = O\left(\frac{nT(n)}{L(n)}\right) = O\left(\frac{n}{\sqrt{L(n)}}\right) = O\left(\frac{n}{\Delta(n)}\right). \tag{16}$$

Focusing our attention on case (i) in Proposition 2[5], we get the following:

Proposition 3: Given $P_{tx}(n) = \frac{\Psi W}{K}\left(\frac{\log n + c(n)}{n}\right)^{\frac{\alpha}{2}}$, with $c(n) = \Theta(\log n)$, the network is connected $\mathbb{P}$-a.s. and its total transport capacity satisfies:

$$S(n) = \Omega\left(\frac{n \log^2(\log n)}{\log^4 n}\right), \tag{17}$$

$$S(n) = O\left(\frac{n}{\log n}\right). \tag{18}$$

Proof: The proof follows substituting the results for case (i), Proposition 2 in (15) and (16). ∎

This result is *independent* of the actual traffic pattern. (Similarly, the case $c(n) = \omega(\log n)$ can also be analyzed, leading to slightly worse performance.)

Let us consider the case in which the network consists of $\frac{n}{2}$ communication pairs, as in [1]. In order to get the result, we will to consider a grid-like tessellation of the torus, as reported in Fig. 3. The unit torus is divided into "squarelets" of area $a(n)$. Under some conditions to be defined later on, each squarelet contains at least one node with probability tending to 1 [23]. The power needs to be scaled so that we can ensure that a node in a given squarelet is able to communicate with all nodes in the eight neighboring squarelets, which translates to:

$$R(n) \geq 2\sqrt{2a(n)}, \tag{19}$$

or, equivalently,

$$P_{tx}(n) \geq \frac{\Psi W}{K}\left[8a(n)\right]^{\frac{\alpha}{2}}. \tag{20}$$

We get the following:

Proposition 4: Given $P_{tx}(n) = \frac{\Psi W}{K}\left(\frac{24 \log n}{n f_{min}}\right)^{\frac{\alpha}{2}}$, the per-connection throughput satisfies:

$$\lambda(n) = \Omega\left(\frac{1}{\sqrt{n \log n}} \frac{\log^2(\log n)}{\log^4 n}\right), \tag{21}$$

$$\lambda(n) = O\left(\frac{1}{\sqrt{n \log n}}\right). \tag{22}$$

[5]The case (ii) is of scarce interest, since in the case of a fully connected, single-hop network, standard CSMA or TDMA techniques could be successfully employed.

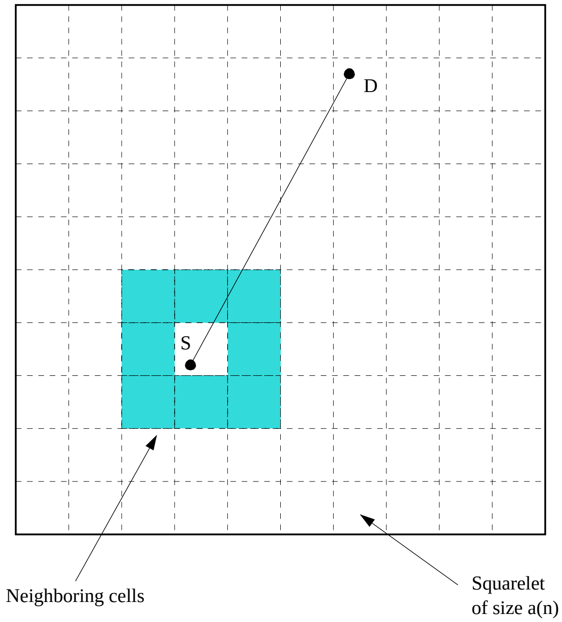

Fig. 3. The grid-like tesselation of the torus into squarelets of size $a(n)$ considered, together with one Sender(S)-Destination(D) pair.

Proof: We start proving the upper bound. Since the average distance between any pair of nodes is $\Theta(1)$, it is easy to see that the average number of hops is $\Theta\left(\frac{1}{R(n)}\right)$. Hence the total traffic to be carried on the network is of the order of $\frac{n\lambda(n)}{R(n)}$, and this has to be smaller than $S(n)$. Using (16), we get (22).
In order to demonstrate the lower bound, we proceed along the lines of [3]. In particular, we consider dividing the network area into "squarelets" of size $a(n) = \frac{3 \ln n}{n f_{min}}$. It can be shown (the derivation is reported in the appendix) that there is $\mathbb{P}$-a.s. at least one node in each squarelet. The scaling chosen for the transmission power satisfies (20) for $a(n) = \frac{3 \ln n}{n f_{min}}$. Further, the network is connected $\mathbb{P}$-a.s. The number of source-destination pairs whose traffic is routed through any chosen squarelet is of the order of $O(n\sqrt{a(n)}) = O\left(nR(n)\right)$. Each squarelet has a "throughput" that is $\Omega\left(\frac{\log^2 \log n}{\log^4 n}\right)$, since it contains at least one node (and the throughput of the squarelet grows with the number of nodes). Each squarelet gets a traffic that is $O\left(n\lambda(n)R(n)\right)$, so that each node can generate traffic at a rate

$$\lambda(n) = \Omega\left(\frac{\log^2 \log n}{nR(n)\log^4 n}\right) = \Omega\left(\frac{1}{\sqrt{n \log n}}\frac{\log^2(\log n)}{\log^4 n}\right).$$

■

This means that, by using TSMA schemes, we are at most a factor $\frac{\log^2(\log n)}{\log^4 n}$ away from the Gupta-Kumar bound.

This result is worth some comments, in that it shows that there exists a distributed mechanism, able to work *without* accurate topology information and to provide a *deterministic* access delay which is able to stay very close (at least in the limiting regime) to the optimal performance in terms of throughput.

In terms of delay, we consider, for the sake of conciseness, a fluid model, as in [23], where the packet size is scaled down with n. The results can be extended to the fixed packet size as in [24]. We get the following:

Proposition 5: Given $P_{tx}(n) = \frac{\Psi W}{K}\left(\frac{24 \log n}{n f_{min}}\right)^{\frac{\alpha}{2}}$, the average packet delay satisfies:

$$D(n) = \Omega\left(\sqrt{n \log n}\right), \tag{23}$$

$$D(n) = O\left(\frac{\log^4 n}{\log^2 \log n}\sqrt{\frac{n}{\log n}}\right). \tag{24}$$

Proof: Again, as in [3], we consider dividing the network area in squarelets of size $a(n) \geq \frac{3 \ln n}{n f_{min}}$. Each squarelet is able to successfully transmit at least once per time frame, so that the (access) delay incurred at each node is of the order of $O(L(n))$. Scaling down packet sizes, all packets are served in the allocated slot subsequent to their arrival [3]. The average delay is thus given by the product of the average number of hops and the average delay incurred at each node. This leads to:

$$D(n) = O\left(\frac{L(n)}{R(n)}\right) = O\left(\frac{\log^4 n}{\log^2 \log n}\sqrt{\frac{n}{\log n}}\right).$$

On the other hand, a lower bound on the access delay is given by $\Omega(\sqrt{L(n)})$, leading to:

$$D(n) = \Omega\left(\frac{\sqrt{L(n)}}{R(n)}\right) = \Omega\left(\sqrt{n \log n}\right).$$

■

Comparing with the optimal tradeoff in [23], we can see that our scheme is at least a factor $\log n$ (and at most a factor $\frac{\log^4 n}{\log^2 \log n}$) from the optimal delay.

From the communication point of view, keeping in mind the scaling we used for the transmission power, we obtain that the only physical-layer parameter the network is sensitive to is the path loss decay factor α, whereas the scaling laws are insensitive (up to a multiplicative constant) to the actual values of the SNR threshold, the noise variance and the antenna gain.

B. Capacity/Delay Tradeoffs in Static Networks

The results in the previous sections can be generalized, as in [23], to study capacity-delay tradeoffs. The basic idea is that, by enlarging the transmission power (and hence the communication range), the delay can be reduced at the expense of a reduced throughput. We get the following:

Theorem 3.1: Given a scaling $P_{tx}(n)$ of the transmission power which satisfies:

$$P_{tx}(n) \geq \frac{\Psi W}{K}\left(\frac{24 \log n}{n f_{min}}\right)^{\frac{\alpha}{2}} \tag{25}$$

and denoting by $R(n)$ and $L(n)$, respectively, the resulting scaling for the communication range and the frame length, the following holds:

$$\lambda(n) = O\left(\frac{R(n)}{\sqrt{L(n)}}\right), \tag{26}$$

$$\lambda(n) = \Omega\left(\frac{1}{nL(n)R(n)}\right), \tag{27}$$

$$D(n) = O\left(\frac{L(n)}{R(n)}\right), \tag{28}$$

$$D(n) = \Omega\left(\frac{\sqrt{L(n)}}{R(n)}\right). \tag{29}$$

Proof: Under the condition (25), the network is connected $\mathbb{P}$-a.s. Further, we can divide the network in squarelets of area

$$a(n) = \frac{1}{8}\left(\frac{KP_{tx}(n)}{\Psi W}\right)^{\frac{2}{\alpha}} \geq 3\frac{\log n}{n}$$

and ensure that transmissions to the neighboring squarelets is always possible. We can then reply the reasoning in Proposition 4 and Proposition 5 to get the results. ■

The comparison with the results (in terms of optimal capacity/delay tradeoffs) in [23] is not straightforward, due to the non-linear dependence of $L(n)$ on $R(n)$.

C. An Extension to the Mobile Case

The results presented in the previous sections can be extended to the case where nodes are mobile. It has indeed been shown that mobility can be exploited to improve network capacity [2] by trading off capacity for delay and storage [3], [5]. The common basis of such schemes is the provisioning of a relaying protocol that is able to exploit the multiuser diversity provided by the nodes' movement to enhance the overall connection throughput.

In this case, since we no longer need to satisfy the connectivity constraint, a faster scaling of the transmission power can be considered. We consider a scaling of the form $P_{tx} = \frac{\Psi W}{K} n^{-\frac{\alpha}{2}}$. While we have no proof of the optimality of such scaling, we used it in conformance with what done in [23]. The resulting transmission range is upperbounded by the ones falling in case (ii) in Proposition 1, which together with (7) leads to a schedule length satisfying:

$$L(n) = O\left(\frac{\log^4 n \log^2 \log\log n}{\log^4 \log n}\right). \tag{30}$$

On the other hand, we used Basagni's lower bound on the frame length [22]:

$$L(n) = \Omega(\log n). \tag{31}$$

Let us now consider a simple i.i.d. mobility model, as in [2] and [3]. Further, we consider Scheme 2 in [23], so that we aim at maximizing throughput at the expense of packet delay. The only difference with the scheme therein is that, in our case, each squarelet transmits at least once every time frame, i.e., every $O\left(\frac{\log^4 n \log^2 \log\log n}{\log^4 \log n}\right)$ seconds. Following their reasoning, this leads to the following lower bound on network throughput:

$$\lambda(n) = \Omega\left(\frac{\log^4 \log n}{\log^4 n \log^2 \log\log n}\right). \tag{32}$$

Again, upper bounds can be found by considering the case in which all transmissions are successful, leading to:

$$\lambda(n) = O\left(\frac{1}{\sqrt{\log n}}\right). \tag{33}$$

In this case, there is a penalty *at least* of the order $\sqrt{\log n}$ for using TSMA schemes.

The computation of the delay follows along the lines in [23], leading to:

$$D(n) = O\left(\frac{n \log^5 n \log^2 \log\log n}{\log^4 \log n}\right); \tag{34}$$

$$D(n) = \Omega\left(n \log^{\frac{3}{2}} n\right). \tag{35}$$

We can then conclude that, also in the mobile case, there is a small penalty in employing TSMA instead of a centralized scheduling scheme.

In Fig. 4 we graphically depicted the results obtained, in terms of throughput and delay, by TSMA schemes, together with the ones attainable by means of a perfect (centralized) scheduling of packet transmissions for both static and mobile networks.

D. Energy Consumption Considerations

One critical issue when dealing with ad hoc networks concerns the energy consumption of such systems. Indeed, since we are talking about battery-driven devices, we should always consider, in the design phase, the node lifetime issue. With respect to such problem, TSMA schemes may offer some advantages over current CSMA-based distributed solutions (e.g., IEEE 802.11). It is indeed known from experimental results that the power consumed for sensing the channel is of the same order of magnitude as that used for transmission. This means that, even in the presence of a low transmitting rate, a large amount of battery power could be drained by the channel sensing mechanism. In general, in typical applications, the energy consumed on performing communication tasks is significantly larger than that used for computational goals. TSMA schemes can then benefit from the absence of channel sensing; on the other hand, they envisage that each node attempts transmitting more than once per frame, with a consequent waste of energy.

For the Chlamtac–Faragó scheme, the number of attempts per frame per node is bounded by:

$$T(n) = O\left(\frac{\Delta(n)\log n}{\log \Delta(n)}\right). \tag{36}$$

We are interested in evaluating the average amount of packets successfully transmitted by a node per unit energy, that we term *energy efficiency* of the protocol and denote by η. For the sake of simplicity, we refer to the static case, even if the analysis can be easily extended to the mobile case, and consider a slot length of 1 second. The energy efficiency is given by the ratio of the number of successful transmissions per frame over the total amount of energy spent per frame. Since there is at least one successful transmission per frame,

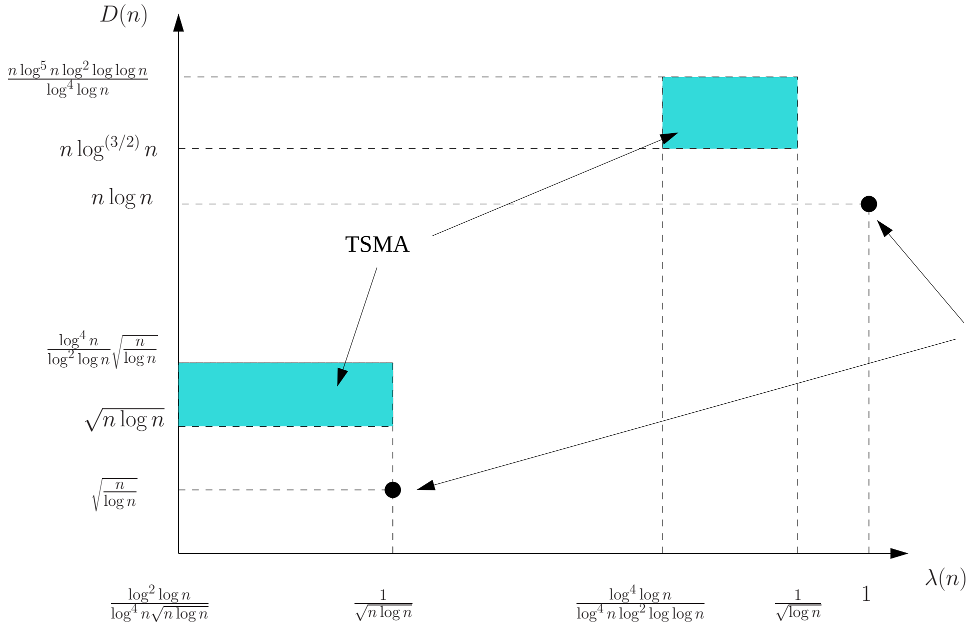

Fig. 4. Graphical representation of the capacity/delay tradeoffs achievable by TSMA in both static and mobile networks and comparison with the optimal values.

we get:

$$\eta(n) = \Omega\left(\frac{1}{P_{tx}(n)T(n)}\right) = \Omega\left(\frac{1}{P_{tx}(n)\sqrt{L(n)}}\right). \quad (37)$$

We can then obtain the following:

Proposition 6: Given $P_{tx}(n) = \frac{\Psi W}{K}\left(\frac{\log n + c(n)}{n}\right)^{\frac{\alpha}{2}}$, with $c(n) = \Theta(\log n)$, the energy efficiency of a TSMA scheme satisfies:

$$\eta(n) = \Omega\left(\frac{n^{\frac{\alpha}{2}}\log\log n}{[\log n]^{\frac{\alpha+4}{2}}}\right). \quad (38)$$

A perfect/centralized scheduling scheme achieves an energy efficiency given by:

$$\eta(n) = \Theta\left(\frac{1}{P_{tx}(n)}\right) = \Theta\left(\frac{n^{\frac{\alpha}{2}}}{[\log n]^{\frac{\alpha}{2}}}\right) \quad (39)$$

Comparing (38) with (39) we can see that in this case we are at most a factor $\frac{\log\log n}{\log^2 n}$ away from the optimal value. The upper bound on the energy efficiency of TSMA can be easily seen to coincide with the optimal value and is therefore of little interest.

IV. Model Extensions and Numerical Results

The model can be extended to include time-varying error-prone channels. In particular, let us assume that the loss process is stationary ergodic and has a "memory" shorter than $L(n)$, the time frame. Hence transmissions in successful frames fail independently with probability p, which denotes the (stationary) probability of packet error events. The time between two successful transmission is thus given by the product of the frame length and a geometric random variable of parameter p, and has mean $\frac{L(n)}{1-p} = \Theta(L(n))$. As a consequence, the scaling laws for capacity and delay remain the same as in the ideal channel case. Clearly, due to the randomness induced by the time-varying channel, the scaling law for the access delay is now given in terms of *average* delay and not deterministic upper-bound on the same quantity.

An important aspect to consider is the network size (in terms of number of nodes) at which the asymptotic behavior starts appearing. Indeed, in real-world deployments, asymptotes are of little interest if they arise only for extremely large values of n. To do so, we run some numerical simulations, generating random topologies and evaluating, for a scaling of the type (iii) in Proposition 1 (in our case $R(n) = \sqrt{\frac{\log n}{n}}$) the ratio of the actual maximum node degree against the one predicted by the asymptotic behavior. Assuming a network area of 100×100 m^2, we reported in Fig. 5 the 95% confidence interval for such ratio for n in the range $100, \ldots, 4100$ nodes. Results, in terms of mean value only, are reported for a wider range, up to 10^5 nodes (in logarithmic scale), in Fig. 6. As it can be seen, the convergence is pretty slow, and even at 10^5 nodes we are not close to the asymptotic behavior. Nonetheless, what is interesting from an application point of view is that the convergence is monotonic from below, which suggests that the asymptotic may be safely used in the dimensioning phase even for small values of n.

The model can also be extended to account for a grid-like topology (as depicted in Fig. 7), which may be of interest for those situations in which it is possible to control the deployment of nodes into the environment (e.g., sensor networks). In such a case, under an appropriate scaling of the

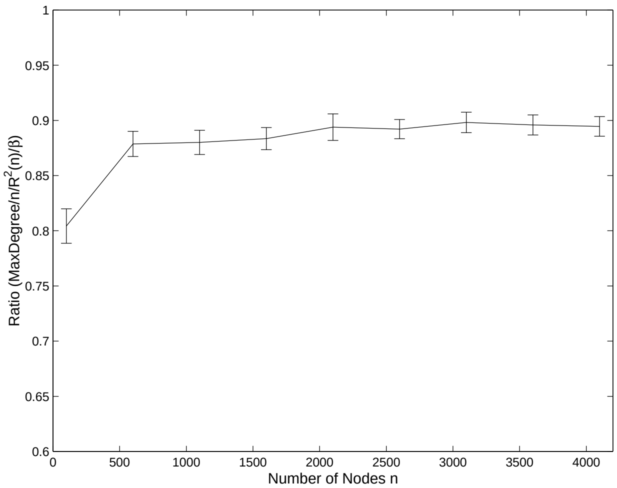

Fig. 5. Ratio of the simulated maximum node degree over the asymptotic value, 95% confidence value over 100 simulations.

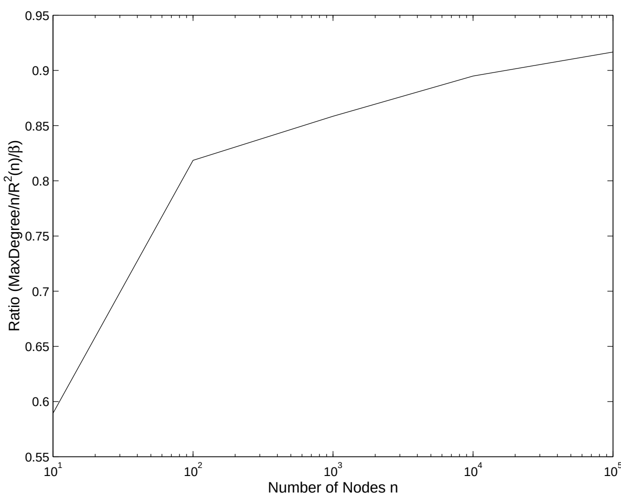

Fig. 6. Ratio of the simulated maximum node degree over the asymptotic value, 95% confidence value over 100 simulations.

transmission power, we can get a connected network while keeping a finite Δ for any n. We only need to ensure that each node is able to communicate with its 4 closest neighbors (placed at a distance $\frac{1}{\sqrt{n}}$), which implies a scaling of the transmission power of the form $P_{tx} = \frac{\Psi W}{K} n^{-\frac{\alpha}{2}}$. Under such conditions, the frame length scales as $O(\log^2 n)$ and $\Omega(\log n)$. We thus obtain the following results:

Proposition 7: Given n nodes deployed in a regular grid over the unit torus, and given a transmission power of the form: $P_{tx} = \frac{\Psi W}{K} n^{-\frac{\alpha}{2}}$, the following holds:

(i) The total network capacity scales as:

$$S(n) = O\left(\frac{n}{\sqrt{\log n}}\right) \tag{40}$$

$$S(n) = \Omega\left(\frac{n}{\log^2 n}\right). \tag{41}$$

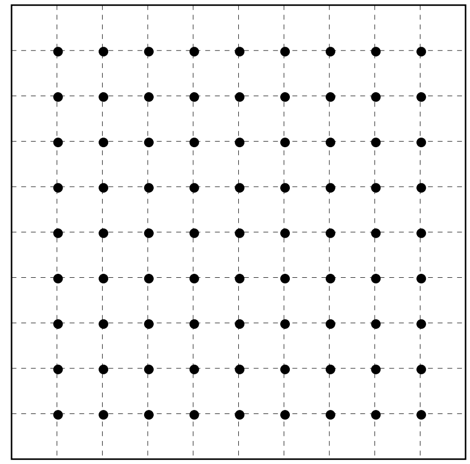

Fig. 7. The grid topology; the distance between neighboring nodes is $\frac{1}{\sqrt{n}}$.

(ii) The per-connection throughput scales as:

$$\lambda(n) = O\left(\frac{1}{\sqrt{n \log n}}\right) \tag{42}$$

$$\lambda(n) = \Omega\left(\frac{1}{\sqrt{n}\log^2(n)}\right). \tag{43}$$

(iii) The average packet delay scales as:

$$D(n) = O\left(\sqrt{n}\log^2 n\right) \tag{44}$$

$$D(n) = \Omega\left(\sqrt{n \log n}\right). \tag{45}$$

(iv) The energy efficiency scales as:

$$\eta(n) = \Omega\left(\frac{n^{\frac{\alpha}{2}}}{\log n}\right). \tag{46}$$

Proof: The proof follows considering the scaling $O(\log^2 n)$ and $\Omega(\log n)$ for $L(n)$ and repeating the reasoning presented for the case of nodes randomly placed on the torus. ■

Note that, with respect to the scaling we used for the random node distribution, here we get a higher throughput and a better energy efficiency at the expense of a higher packet delay. Choosing the scaling for $P_{tx}(n)$ used in Sec. III would lead to the same results obtained for the random topology, since all results depend just on the scaling of the frame length. It is worth noting that, in terms of throughput, we gain a factor $\sqrt{\log n}$ over the random case. A similar result is also in [1]. It remains an open question for future studies the possibility of closing such gap along the lines of what proposed in [17].

V. Conclusions

In this paper, we have studied the asymptotic behavior of an ad hoc network employing a topology-transparent scheduling algorithm. Using some limiting results from the theory of Geometric Random Graphs, we have characterized the asymptotic behavior of the maximum node degree for various regimes of the communication range. Such results have been used to derive the limiting behavior of the frame length in a Chlamtac-Faragó TSMA scheme. From that, scaling laws for the capacity and delay have been derived. The extension of the proposed analytical framework to the case of mobile nodes has been presented, and a characterization of the energy efficiency of TSMA protocols has been introduced and discussed.

There are two directions we are currently pursuing for extending the presented results. The first one deals with an attempt to close down (or at least refine) the bounds obtained for the frame length (and consequently for capacity and delay). The second one deals with the impact of "spatial" channel impairments, when the communication range is not any longer a fixed value but is a random variable whose distribution may account for the impact of shadowing and fading, as in [21].

Appendix

We prove that, under our assumptions on the node distribution, if $a(n) \geq 3\frac{\log n}{n f_{min}}$, there is $\mathbb{P}$-a.s. at least one node per squarelet. The proof follows along the lines of [25]. Let $m = \frac{1}{a(n)}$ be the number of squarelets. For each squarelet i, we denote by A_i the event that there are no nodes in i. Since the probability of not having a given node in squarelet i is upperbounded by $1 - \frac{f_{min}}{m}$, we have:

$$\mathbb{P}[A_i] \leq \left(1 - \frac{f_{min}}{m}\right)^n .$$

By the union bound, the probability that at least one square is empty is upperbounded by:

$$p_n = m\left(1 - \frac{f_{min}}{m}\right)^n .$$

Since $1 - x \leq e^{-x}$,

$$p_n \leq m e^{-\frac{f_{min} n}{m}} \leq \frac{n}{3 f_{min} \log n} e^{3 \log n} = \frac{1}{3n^2 f_{min} \log n} \leq \frac{1}{n^2} .$$

Since $\sum_{n=1}^{+\infty} p_n < +\infty$, using the Borel-Cantelli lemma we conclude that almost surely no squarelet is empty for sufficiently large n.

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