Trading Dynamics with Private Buyer Signals in the Market for Lemons

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Abstract

We present a dynamic model of trading under adverse selection in which a seller sequentially meets buyers, each of whom receives a noisy signal about the quality of the asset and offers a price. We fully characterize the equilibrium trading dynamics and show that buyers’ beliefs about the quality of the asset can both increase or decrease over time, depending on the initial level. This result demonstrates how the introduction of private buyer signals enriches the set of trading patterns that can be accommodated within the framework of dynamic adverse selection, thereby broadening its applicability. We provide various applications and empirical implications of our model and discuss the robustness of our main insights in multiple dimensions.

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1 Introduction

Buyers often draw inferences about the quality of an asset (good) from its duration on the market. In the real estate market, a long time on the market is typically interpreted as bad news (see, e.g., Taylor, 1999; Tucker, Zhang and Zhu, 2013). This is arguably the reason why some sellers reset

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their days on the market by relisting their properties without undertaking any major repairs or renovations.\footnote{This is a common practice, but its harmful effects are well-recognized. Blanton (2005) criticizes that it is like “resetting the odometer on a used car.” The real estate listing service in Massachusetts decided to prevent the practice in 2006.} In the labor market, unemployment duration affects a worker’s reemployment probability and reservation wage (see, e.g., Lynch, 1989; Oberholzer-Gee, 2008). This duration dependence is often attributed to the so-called “non-employment stigma,” which refers to the phenomenon that employers tend to interpret a long unemployment spell as a bad signal about the worker’s productivity and are more reluctant to hire such a worker. Interestingly, empirical findings for these intuitive observations are mixed. Some studies find no relationship between time-on-the-market and sale price or employment probability (see, e.g., Belkin, Hempel and McLeavey, 1976; Lancaster, 1979; Heckman and Borjas, 1980). Some others even report opposite patterns: a positive relationship between time-on-the-market and sale price, and increasing reemployment probabilities over unemployment spells (see, e.g., Miller, 1978; Heckman and Singer, 1984; Butler and McDonald, 1986).

The goal of this paper is to understand when, and why, delay is perceived as good or bad news about the quality of an asset. The answer ultimately depends on the source of delay. Among many factors that contribute to delay, the following three seem particularly prominent:\footnote{Each of the three sources has been extensively studied in the literature: the first in the literature on sequential search, going back to Stigler (1961); the second in the growing literature on dynamic adverse selection (see Evans, 1989; Vincent, 1988; 1990; Janssen and Roy, 2002; Deneckere and Liang, 2006; Hörmann and Vieille, 2009; Moreno and Wooders, 2010, for some seminal contributions); and the last in the literature on unemployment duration dependence and, more broadly, observational learning (see Vishwanath, 1989; Lockwood, 1991; Banerjee, 1992; Bikhchandani, Hirshleifer and Welch, 1992, for some seminal contributions).} First, delay could be just because of search frictions, that is, a seller may have been unlucky and not met any buyer yet. If this is the main source for delay, buyers’ inferences about the quality of an asset should be independent of the seller’s time-on-the-market. Second, delay might be caused by adverse selection. A high-quality seller, due to her higher reservation value, is more willing to wait for a high price than a low-quality seller. In this case, delay conveys good news about the quality of the asset. Finally, previous buyers might have decided not to purchase after observing an unfavorable attribute. If this is the main driving force for delay, delay is interpreted as bad news and buyers get more pessimistic about the quality of the asset over time. We build a model that incorporates these three sources and study how they are linked to one another and to other aspects of the market environment.

Specifically, we consider the problem of a seller who possesses an indivisible asset and sequentially meets buyers. There are two types of assets, low quality and high quality. There are always gains from trade, but the quality of the asset is known only to the seller. Each buyer receives a noisy signal about the quality of the asset and makes a price offer. Buyers’ signals are private (i.e.,
not observable by other buyers), but each buyer observes the seller’s time-on-the-market (i.e., how long the asset has been up for sale).

We show that whether delay is good or bad news depends on an asset’s initial reputation (i.e., buyers’ prior beliefs about the quality of the asset). If an asset enjoys a rather high reputation initially, the asset’s reputation, conditional on no trade, declines over time, while if an asset starts out with a low reputation, then the asset’s reputation improves over time. To understand these opposing patterns, first note that the higher an asset’s reputation is, the more likely buyers are to offer a high price. This implies that while enjoying a high reputation, even a low-quality seller would choose to hold out for a future chance of a high price and, therefore, be reluctant to accept a low price. In this case, trade can be delayed only when, despite the asset’s high reputation, buyers are unwilling to offer a high price, which is the case when they receive sufficiently unfavorable inspection outcomes. Since a low-quality asset is more likely to generate such inspection outcomes, the asset is deemed less likely to be of high quality, the longer it stays on the market. In the opposite case when an asset suffers from a low reputation, a low-quality seller would be willing to settle for a low price, while a high-quality seller would still insist on a high price in order to recoup his higher cost. Since a high-quality asset would stay on the market relatively longer than a low-quality asset, the asset’s reputation improves over time.

Our result broadens the applicability of the theory of dynamic adverse selection. The literature on dynamic adverse selection is growing fast, mainly because it can provide a synthetic theory of several forms of market inefficiencies and be used to address various policy issues, including the policies that have been implemented or stipulated after the recent financial crisis. A common theoretical insight of this literature is that delay can be used as a signaling device and, therefore, an asset’s reputation necessarily improves over time. This is precisely the main working mechanism in our model when an asset’s initial reputation is low. Our contribution is, by incorporating private buyer signals, to accommodate the possibility that an asset’s reputation deteriorates over time within the framework of dynamic adverse selection. As introduced at the beginning, delay is perceived as bad news in several markets. Our result explains how, and when, such a pattern can arise in models of dynamic adverse selection.

The role of search frictions in equilibrium trading dynamics deserves elaboration. First, although search frictions are neutral to the direction of the evolution of beliefs, they affect the speed of the evolution. Buyers can never exclude the possibility that the seller has been so unfortunate that no buyer has contacted her yet. This forces buyers’ beliefs to change gradually. Second, they indirectly influence the direction of the evolution of beliefs through their impact on the equilibrium structure. In particular, a reduction in search frictions makes the decreasing pattern more prevalent: the threshold initial reputation level decreases as search frictions reduce. Finally, search frictions are responsible only for a portion of delay: even if search frictions are arbitrarily small,
the expected time to trade remains bounded away from zero. This is similar to the persistence of delay in other models of dynamic adverse selection, but differs in that it holds despite the fact that each buyer generates a constant amount of information and, therefore, an arbitrarily large amount of *exogenous* information is instantaneously generated about the quality of the asset in the search-frictionless limit.

Our model, either by direct comparative statics (Sections 4 and 5) or with some simple extensions (Section 6), allows us to address various questions that are of applied interest and may be relevant for actual government policies. One particularly intriguing question is the effects of varying the informativeness of buyers’ signals. It is widely accepted that asset (corporate) transparency improves market efficiency by facilitating socially desirable trade. Such beliefs have been reflected in recent government policies, such as the Sarbanes-Oxley Act passed in the aftermath of the Enron scandal and the Dodd-Frank Act passed in the aftermath of the recent financial crises, both of which include provisions for stricter disclosure requirements on the part of sellers. Presumably, the main goal of such policies is to help buyers assess the merits and risks of financial assets more accurately. This naturally corresponds to an increase in the informativeness of buyers’ signals in our model.

We demonstrate that enhancing asset transparency does not necessarily lead to efficiency improvement. We show that an increase in the informativeness of buyers’ signals, in the sense of Blackwell (1951), may slow down trade of both types and be harmful to some sellers and buyers. In addition, with a straightforward modification of our model, we demonstrate that if some buyers are uninformed (i.e., do not possess the inspection technology), then increasing the informativeness of informed buyers’ signals may have an indirect negative effect on uninformed buyers, exacerbating the inequality between the two groups. These results arise mainly because of the strategic influence each (informed) buyer’s signal has on other buyers’ inferences. When an asset’s reputation is expected to decrease over time, an increase in the informativeness of buyers’ signals speeds up the decline, resulting in late buyers becoming even more pessimistic about the quality of the asset. This reduces buyers’ incentives to offer a high price, thereby slowing down trade.

**Related Literature**

Most existing studies on dynamic adverse selection focus on the implications of the difference in different types’ reservation values and, therefore, feature one form of equilibrium dynamics in which low-quality assets trade faster. One notable exception is Taylor (1999). He studies a two-period model in which the seller runs a second-price auction with a random number of buyers in each period and the winner conducts an inspection, which generates a bad signal only when the quality is low. He considers several settings that differ in terms of the observability of first-
period trading outcomes (in particular, inspection outcome and reservation (list) price history) by second-period buyers. In all settings, buyers assign a lower probability to the high quality in the second period than in the first period (that is, buyers’ beliefs decrease over time). Despite various differences in modeling, the logic behind the evolution of beliefs is similar to ours: trade occurs only when the winner receives a good signal, and the high type is more likely to generate a good signal than the low type. Therefore, the asset remaining in the second period is more likely to be the low type. However, the opposite form of trading dynamics (i.e., buyers’ beliefs increase over time) is absent in his model. In addition, he addresses various other economic problems, such as the dynamics of reserve (list) prices and the effects of the observability of first-period reserve price and inspection outcome, while we focus on better understanding equilibrium trading dynamics.

Two papers consider an environment similar to ours. Lauermann and Wolinsky (2015) investigate the ability of prices to aggregate dispersed information in a setting where, just like in our model, an informed player (buyer in their model) faces an infinite sequence of uninformed players, each of whom receives a noisy signal about the informed player’s type. Using a similar model with an additional feature that the informed player can contact only a finite number of uninformed players. Zhu (2012) makes a number of interesting observations regarding trading in opaque over-the-counter markets. In both studies, in contrast to our model, uninformed players have no access to the informed player’s trading history. In particular, uninformed players do not observe the informed player’s time-on-the-market. This induces uninformed players’ beliefs and strategies to be necessarily stationary (i.e., their beliefs do not evolve over time). To the contrary, the evolution of uninformed players’ beliefs and the resulting trading dynamics are the main focus of this paper.

Daley and Green (2012) study the role of exogenous information (“news”) about the quality of an asset in a setting similar to ours. The most crucial difference from ours is that news is public information to all buyers. This implies that buyers do not face an inference problem regarding other buyers’ signals, making their trading dynamics quite distinct from ours. Similarly to us, they also explore the effects of increasing the quality of news and find that it is not always efficiency-improving. However, the mechanism leading to the conclusion is quite different from ours. In particular, the negative effect of increased informativeness stems from buyers’ inferences about other buyers’ signals in our model, while in Daley and Green (2012), it is due to its impact on the incentive of the high-type seller to wait for good news.

The rest of the paper is organized as follows. We formally introduce the model in Section 2 and provide a full characterization in Section 3. We study the role of search frictions in Section 4 and analyze the effects of changing the informativeness of buyers’ signals in Section 5. In Section

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3This is due to his assumption that there are no gains from trade of a low-quality asset. In this case, buyers have no incentive to offer a price that can be accepted only by the low type, and thus the low type can not trade faster than the high type. In Appendix A, we consider the comparable case and show that the same result holds in our model.
In Section 6, we explain how our model can be used to address various applied problems. In Section 7, we demonstrate the robustness of our main insights in three dimensions: the number of seller types, the bargaining protocol, and the market structure. In Section 8, we conclude by providing several empirical implications and suggesting various directions for future research.

2 The Model

2.1 Physical Environment

A seller wishes to sell an indivisible asset. Time is continuous and indexed by \( t \in \mathbb{R}_+ \). The time the seller comes to the market is normalized to 0. Potential buyers arrive sequentially according to a Poisson process of rate \( \lambda > 0 \). Once a buyer arrives, he receives a private signal about the quality of the asset and offers a price to the seller. If the seller accepts the price, then they trade and the game ends. Otherwise, the buyer leaves, while the seller waits for the next buyer. The seller discounts future payoffs at rate \( r > 0 \).

The asset is either of low quality (\( L \)) or of high quality (\( H \)). If the asset is of low quality, the seller obtains flow payoff \( r_{cL} \) from owning the asset, while a buyer, once he acquires it, receives flow payoff \( rv_L \). The corresponding values for high quality are \( r_{cH} \) and \( rv_H \), respectively. There are always gains from trade: \( c_L < v_L \) and \( c_H < v_H \). However, the quality of the asset is private information of the seller. It is commonly known that the asset is of high quality with probability \( \hat{q} \) at time 0.

Each buyer’s signal \( s \) is drawn from a finite set \( S = \{s_1, ..., s_N\} \). If the quality is low (respectively, high), then the probability that a buyer receives signal \( s_n \) is given by \( \gamma_L(s_n) \) (respectively, \( \gamma_H(s_n) \)). Without loss of generality, we assume that the likelihood ratio \( \gamma_H(s_n) / \gamma_L(s_n) \) is strictly increasing in \( n \), so that the higher a signal is, the higher posterior probability each buyer assigns to the asset being of high quality. For later use, denote by \( \Gamma_a(s_n) \) (respectively, \( \Gamma_a^{-}(s_n) \)) the probability that a buyer receives a signal weakly (respectively, strictly) below \( s_n \) from a type-\( a \) asset (i.e., \( \Gamma_a(s_n) \equiv \sum_{n' \leq n} \gamma_a(s_{n'}) \), and \( \Gamma_a^{-}(s_n) \equiv \sum_{n' < n} \gamma_a(s_{n'}) \), for each \( a = L, H \)).

For most parts of the paper, we restrict attention to the case where search frictions are not particularly large. Formally, we make use of the following assumption:

Assumption 1

\[ v_L < \int_0^\infty \left[ (1 - e^{-rt})c_L + e^{-rt}c_H \right] d(1 - e^{-\lambda t}) = \frac{r c_L + \lambda c_H}{r + \lambda} \Leftrightarrow r(v_L - c_L) < \lambda(c_H - v_L). \]

The assumption states that the low-type seller has no incentive to accept buyers’ maximal willingness-to-pay for a low-quality asset \( v_L \), should she expect to receive at least the high-type seller’s reser-
vation value \((c_H)\) from the next buyer. We explain what happens if this assumption is violated in Appendix A. We note that \(c_H > v_L\) is necessary for Assumption 1 to hold, but a stronger assumption commonly adopted in the literature, \(q v_H + (1 - q) v_L < c_H\), is not. In fact, Assumption 1 is independent of the initial belief \(\hat{q}\).

We assume that buyers observe (only) how long the asset has been up for sale (i.e., time \(t\)).

This gives tractability to our analysis of non-stationary trading dynamics under adverse selection. It also has a notable technical advantage. For any \(t\), there is a positive probability \((e^{-\lambda t})\) that no buyer has arrived and, therefore, trade has not occurred by time \(t\). This means that there are no off-equilibrium-path public histories and, therefore, buyers’ beliefs at any point in time can be obtained through Bayesian updating.

2.2 Strategies and Equilibrium

The offer strategies of buyers are represented by a function \(\sigma_B : \mathbb{R}_+ \times S \times \mathbb{R}_+ \rightarrow [0, 1]\), where \(\sigma_B(t, s, p)\) denotes the probability that the buyer who arrives at time \(t\) and receives signal \(s\) offers price \(p\) to the seller. The offer acceptance strategy of the seller is represented by a function \(\sigma_S : \{L, H\} \times \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow [0, 1]\), where \(\sigma_S(a, t, p)\) denotes the probability that the type-\(a\) seller accepts price \(p\) at time \(t\). An outcome of the game is a tuple \((a, t, p)\), where \(a\) denotes the seller’s type, \(t\) represents the time of trade, and \(p\) is the transaction price. All agents are risk neutral. Given an outcome \((a, t, p)\), the seller’s payoff is given by \((1 - e^{-rt})c_a + e^{-rt}p\). The buyer who trades with the seller receives \(v_a - p\), while all other buyers obtain zero payoff.

We study perfect Bayesian equilibria of this dynamic trading game. Let \(q(t)\) represent buyers’ beliefs that the seller who has not traded until \(t\) is the high type. In other words, \(q(t)\) is the belief held by the buyer who arrives at time \(t\) prior to his inspection. A tuple \((\sigma_S, \sigma_B, q)\) is a perfect Bayesian equilibrium of the game if the following three conditions hold:

(i) Buyer optimality: \(\sigma_B(t, s, p) > 0\) only when \(p\) maximizes the expected payoff of the buyer with signal \(s\) at time \(t\), that is,

\[
p \in \arg\max_p q(t)\gamma_H(s)\sigma_S(H, t, p')(v_H - p') + (1 - q(t))\gamma_L(s)\sigma_S(L, t, p')(v_L - p').
\]

4It is well-known that the information buyers have about past histories of the game plays a crucial role in this type of game. See Nöldeke and van Damme (1990), Swinkels (1999), Hörner and Vieille (2009), Kim (2014), Kaya and Liu (2014), and Fuchs, Öry and Skrzypacz (2014).

5In principle, the seller’s strategy could depend on her private history, such as the number of previous buyers and rejected offers. This would enlarge the set of equilibria by allowing the seller to use her private history as a randomization device. However, all such equilibria would be essentially equivalent to the one in which the seller’s strategy is independent of her private history, as buyers’ strategies depend only on the seller’s observable characteristic (time-on-the-market).
(ii) Seller optimality: \( \sigma_S(a, t, p) > 0 \) only when \( p \) is weakly greater than the type-\( a \) seller’s continuation payoff at time \( t \), that is,

\[
p \geq E_{\tau, p'} \left[ \left(1 - e^{-r(\tau-t)} \right) c_a + e^{-r(\tau-t)} p' \big| a, t \right],
\]

where \( \tau \) and \( p' \) denote the random time and price, respectively, at which trade takes place according to the strategy profile \((\sigma_S, \sigma_B)\).

(iii) Belief consistency: \( q(t) \) is derived through Bayes’ rule, that is,

\[
q(t) = \frac{\hat{q} e^{-\lambda t} \int_{H} \gamma_H(s) \left( \int f_{\sigma_B(x, s, p) \sigma_S(H, x, p) dp} \right) dx}{\hat{q} e^{-\lambda t} \int_{H} \gamma_H(s) \left( \int f_{\sigma_B(x, s, p) \sigma_S(H, x, p) dp} \right) dx + (1 - \hat{q}) e^{-\lambda t} \int_{L} \gamma_L(s) \left( \int f_{\sigma_B(x, s, p) \sigma_S(L, x, p) dp} \right) dx}.
\]

Let \( p(t) \) denote the low-type seller’s reservation price (i.e., the price which the low-type seller is indifferent between accepting and rejecting) at time \( t \). We restrict attention to strategy profiles in which each buyer offers either \( c_H \) or \( p(t) \). This restriction incurs no loss of generality. First, for the same reasoning as in the Diamond paradox, buyers never offer a price strictly above \( c_H \). This implies that the high-type seller’s reservation price is always equal to her reservation value of the asset \( c_H \), and in equilibrium she accepts \( c_H \) with probability 1. Notice also that, due to the difference in flow payoffs, the low-type seller’s reservation price is always strictly smaller than that of the high type (i.e., \( p(t) \leq \int_{0}^{\infty} \left( (1 - e^{-rt}) c_L + e^{-rt} c_H \right) d(1 - e^{-\lambda t}) < c_H \) for any \( t \)). Second, it is strictly suboptimal for a buyer to offer a price strictly above \( c_H \) or between \( p(t) \) and \( c_H \). Finally, if in equilibrium a buyer offers a losing price (strictly below \( p(t) \)), then it suffices to set his offer to be equal to \( p(t) \) and the low type’s acceptance strategy \( \sigma_S(L, t, p(t)) \) to reflect her rejection of the buyer’s losing offer.

6Formally, let \( \overline{p} \) denote the supremum among all equilibrium prices buyers offer in this game. Suppose \( \overline{p} \geq c_H \). Then, the best case scenario for the high-type seller is to receive \( \overline{p} \) with probability 1 from the next buyer. This means that her reservation price at any point in time cannot strictly exceed

\[
\int_{0}^{\infty} \left[ (1 - e^{-rt}) c_H + e^{-rt} \overline{p} \right] d(1 - e^{-\lambda t}) = \frac{rc_H + \lambda \overline{p}}{r + \lambda}.
\]

This implies that no buyer has an incentive to offer a price above \( (rc_H + \lambda \overline{p})/(r + \lambda) \), and thus \( \overline{p} \leq (rc_H + \lambda \overline{p})/(r + \lambda) \). On the other hand, due to discounting (i.e., \( r > 0 \)) and search frictions (i.e., \( \lambda < \infty \)), \( (rc_H + \lambda \overline{p})/(r + \lambda) \leq \overline{p} \). Therefore, it must be that \( \overline{p} = c_H \). Intuitively, due to discounting and search frictions, each buyer possesses a temporary monopsony power. Whenever \( \overline{p} > c_H \), each buyer can undercut the price to \( (rc_H + \lambda \overline{p})/(r + \lambda) \) and still make sure that the offer is accepted. Knowing that no buyer would offer \( \overline{p} \) and the highest price offer would be \( (rc_H + \lambda \overline{p})/(r + \lambda) \), each buyer can undercut the price even further. This process continues indefinitely as long as \( \overline{p} > c_H \). Therefore, in equilibrium \( \overline{p} \) cannot exceed \( c_H \).
3 Equilibrium Characterization

In this section, we characterize the generically unique equilibrium of the dynamic trading game. We first illustrate the equilibrium structure and present our main characterization result. We then describe the resulting trading dynamics and formally construct the generically unique equilibrium. We also discuss our generic uniqueness result.

3.1 Equilibrium Structure

\[ q_0 = 0 \quad \bar{q}_5 \quad \bar{q}_4 \quad \bar{q}_3 = q^* \quad \bar{q}_2 \quad \bar{q}_1 \quad \bar{q}_0 = 1 \]

| offer \( c_H \): | never | \{s_5\} | \{s_4, s_5\} | \{s_3, s_4, s_5\} | \{s_2, ..., s_5\} | always |
|------------------|--------|--------|-----------|-----------------|--------|
| \( p(t) : \)     | \( < v_L \), \( L \) accepts \( p(t) \) | \( > v_L \), \( L \) rejects \( p(t) \) | \( = \frac{rc_L + \lambda c_H}{r + \lambda} \) |
| \( q(t) : \)     | increase | decrease | constant |

Figure 1: Equilibrium structure when there are 5 signals (\( N = 5 \)) and \( q^* = \bar{q}_3 \).

Figure 1 depicts a typical equilibrium structure in an example with 5 signals. The first row represents buyers’ offer strategies as a function of their beliefs: if \( q(t) \in (\bar{q}_{n+1}, \bar{q}_n) \), then buyers offer \( c_H \), as opposed to \( p(t) \), only when they receive a higher signal than \( s_n \). The interval-partitional structure is intuitive, because buyers’ willingness to offer \( c_H \) depends on their posterior beliefs about the seller’s type, which are increasing in both their prior beliefs and signal. The second row summarizes the low-type seller’s reservation price and equilibrium acceptance strategy. Her reservation price is increasing in buyers’ beliefs (i.e., the more optimistic buyers are about her type, the higher expected payoff she obtains). Since \( p(t) \) can be accepted only by the low type, trade at \( p(t) \) occurs only when \( p(t) \leq v_L \), which is when buyers’ beliefs are below \( q^* = \bar{q}_3 \). The last row describes how buyers’ beliefs \( q(t) \) evolve over time. Unless \( \tilde{q} \) is so high that all buyers offer \( c_H \) regardless of their signal, \( q(t) \) converges to \( q^* \), whether starting from above or below \( q^* \), and stays constant thereafter.

We present our main characterization of agents’ equilibrium strategies in the following theorem. We explain the resulting trading dynamics (in particular, how the convergence to \( q^* \) occurs) in detail in Section 3.2.

Theorem 1 For a generic set of parameter values, there exist a unique finite partition \( \{\bar{q}_{N+1} = 0, \bar{q}_N, ..., \bar{q}_1, \bar{q}_0 = 1\} \) and a unique integer \( n^*(\leq N) \) such that any equilibrium strategy profile
exhibits the following properties:

- If \( q(t) \in (\bar{q}_n, \bar{q}_1) \), then buyers offer \( c_H \) when they receive a signal strictly above \( s_n \) and offer \( p(t) \) otherwise.

- If \( q(t) \in (\bar{q}_n, \bar{q}_1) \) for some \( n < n^* \), then the low-type seller’s reservation price \( p(t) \) exceeds \( v_L \), and she accepts only \( c_H \).

- If \( q(t) \in (\bar{q}_n, \bar{q}_1) \) for some \( n \geq n^* \), then the low-type seller’s reservation price \( p(t) \) falls short of \( v_L \), and she accepts both \( c_H \) and \( p(t) \).

- Suppose \( q(t) = \bar{q}_n \) and denote by \( \tilde{s}_n \) the signal a buyer receives.
  - The buyer offers \( c_H \) with probability 1 if \( \tilde{n} > n^* \), with probability \( \sigma_{B}^* \in (0, 1) \) if \( \tilde{n} = n^* \), and with probability 0 if \( \tilde{n} < n^* \).
  - The low-type seller’s reservation price \( p(t) \) is equal to \( v_L \), and she accepts \( p(t) \) with probability \( \sigma_{S}^* \).
  - \( \sigma_{B}^* \) and \( \sigma_{S}^* \) are determined so that both \( q(t) \) and \( p(t) \) stay constant.

The equilibrium outcome is essentially unique: equilibria vary only with respect to the behavior of buyers who arrive when \( q(t) = \bar{q}_n \) for some \( n \neq n^* \). In all equilibria, however, such events have zero probability, because \( q(t) \) is strictly monotone unless \( q(t) \geq \bar{q}_1 \) or \( q(t) = q^* \). We further discuss this indeterminacy and its essential irrelevance in Section 3.2.

In what follows, for notational convenience, we use \( q^* \) to denote \( q_n^* \). In addition, we denote by \( \tilde{p}(q) \) the low-type seller’s reservation price when buyers’ beliefs are given by \( q \).

### 3.2 Equilibrium Trading Dynamics

Taking as given the partition \( \{\bar{q}_{N+1} = 0, \bar{q}_N, \ldots, \bar{q}_1, \bar{q}_0 = 1\} \) informing buyers’ offer behavior and the corresponding acceptance strategy of the seller as described in Theorem 1, we explain how buyers’ beliefs \( q(t) \) and the low-type seller’s reservation price \( p(t) \) evolve over time in our model.

#### 3.2.1 Evolution of Beliefs

We first illustrate how buyers’ beliefs \( q(t) \) evolve over time. There are two cases when buyers’ beliefs do not vary over time. First, if the probability of the high type is sufficiently large (precisely, \( q(t) > \bar{q}_1 \)), then buyers offer \( c_H \) regardless of their signal. In this case, delay is caused only by search frictions and both seller types trade at an identical rate. Second, as stated in the theorem, \( q(t) \) stays constant once it reaches \( q^* = \bar{q}_n^* \). We formally derive the unique value of \( q^* \) and the corresponding equilibrium strategies in Section 3.3.1.
Figure 2: Evolution of buyers’ beliefs in an example with 5 signals and \( n^* = 3 \).

If \( q(t) \in (q^*, q_1) \), then \( q(t) \) strictly decreases over time (see the solid line in Figure 2). Its falling speed depends on the current level of \( q(t) \) and, therefore, varies over time. If \( q(t) \in (q_{n+1}, q_n) \) for some \( n < n^* \), then buyers offer \( c_H \) if and only if they receive a signal strictly above \( s_n \), and both seller types accept only \( c_H \). This implies that the trading rate of the type-\( a \) seller is given by \( \lambda(1 - \Gamma_a(s_n)) \) for each \( a = H, L \), and \( q(\cdot) \) evolves according to the following law of motion:

\[
\dot{q}(t) = q(t)(1 - q(t))\lambda(\Gamma_H(s_n) - \Gamma_L(s_n)) < 0. \tag{1}
\]

This expression is strictly negative, because \( \Gamma_H(s_n) < \Gamma_L(s_n) \) for any \( n < N \) (first-order stochastic dominance). Intuitively, in this region trade occurs only at \( c_H \), while buyers offer \( c_H \) only when they receive a sufficiently good signal. Therefore, a seller who fails to trade and stays longer is more likely to be the low type, who generates good signals less frequently than the high type.

To the contrary, if \( q(t) < q^* \), then \( q(t) \) strictly increases over time (see the dashed line in Figure 2). Fix \( n \geq n^* \), and suppose \( q(t) \in (q_{n+1}, q_n) \). In this case, the trading rate of the high-type seller is, again, equal to \( \lambda(1 - \Gamma(s_n)) \), while that of the low-type seller is equal to \( \lambda \); the low-type seller accepts not only \( c_H \) but also \( p(t) \), and thus her trading rate is equal to the arrival rate of buyers.

\[\text{Heuristically, by Bayes’ rule,}\]

\[
q(t + dt) = \frac{q(t)e^{-\lambda(1 - \Gamma_H(s_n))\lambda dt}}{q(t)e^{-\lambda(1 - \Gamma_H(s_n))\lambda dt} + (1 - q(t))e^{-\lambda(1 - \Gamma_L(s_n))\lambda dt}}.
\]

Equation (1) can be derived by subtracting \( q(t) \) from both sides and dividing by \( dt \).
The law of motion for $q(\cdot)$ is then given as follows:

$$
\dot{q}(t) = q(t)(1 - q(t))\lambda \Gamma_H(s_n) > 0.
$$

(2)

This is when the seller’s reputation is relatively low and, therefore, buyers are reluctant to offer $c_H$. In this case, the low-type seller prefers to settle on a reasonable price early, rather than insisting on a high price for a long time. Therefore, a seller who stays longer is more likely to be the one who suffers less from waiting, which is the high type.

Figure 2 depicts two typical paths of buyers’ beliefs in an example with 5 signals and $n^* = 3$. If buyers’ beliefs start from $\hat{q} \in (\bar{q}_2, \bar{q}_1)$, then they decrease according to equation (1) with $s_n = s_1$. The cutoff signal changes from $s_1$ to $s_2$ when $q(t) = \bar{q}_2$, but buyers’ beliefs still decline over time. Once buyers’ beliefs hit $q^* = \bar{q}_{n^*}$, they stay constant thereafter. The interpretation of the dashed line is identical, except that buyers’ beliefs increase whenever they are below $q^*$.

Notice that if there is only one signal (so effectively no informative signal), then it is necessarily the case that $q^* = \bar{q}_1$. Therefore, the decreasing belief dynamics cannot exist: buyers’ beliefs either stay constant (if above $q^*$) or strictly increase over time (if below $q^*$). This explains the role of private buyer signals in our model and pinpoints our unique contribution to the literature on dynamic adverse selection. Most existing models feature only weakly increasing belief dynamics (e.g., Hörner and Vieille, 2009; Camargo and Lester, 2014; Kim, 2014). Our result shows that the introduction of private buyer signals enriches the set of trading patterns that can be accommodated within the framework of dynamic adverse selection.

In Theorem 1, we do not specify buyers’ offer strategies when their beliefs are equal to $\bar{q}_n$ for $n \neq n^*$. As remarked right after the theorem, they are in fact not uniquely pinned down: if a buyer has prior belief $\bar{q}_n$ and receives signal $s_n$, then he is indifferent between offering $c_H$ and $p(t)$. The offer strategies at such points, however, do not affect the equilibrium outcome, because $q(t)$ is strictly monotone and the arrival rate of buyers is finite, and thus the probability that a buyer arrives exactly at the point when his belief is equal to one of the cutoffs is equal to 0.

### 3.2.2 (Reservation) Price Dynamics

We now illustrate how prices vary over time in our model. Since the high-type seller’s reservation price is always equal to $c_H$ and each buyer offers either $c_H$ or $p(t)$ (with $p(t)$ possibly being a losing offer), we focus on the low-type seller’s reservation price $p(t)$.

One useful observation is that $p(t)$ is determined only by the rate at which the low type receives offer $c_H$: each buyer offers either $c_H$ or $p(t)$, and the low type is indifferent between accepting and rejecting $p(t)$. Therefore, for the purpose of calculating her expected payoff, we can assume that she accepts only $c_H$. Note that this does not mean that in equilibrium the low-type seller never
Figure 3: The rate at which the low-type seller receives offer $c_H$ (left) and her reservation price (right) as functions of time $t$. For the definition of $\rho_L$, see Section 3.3.1.

accepts $p(t)$. She does accept $p(t)$ if $q(t) < q^*$. However, the seller’s rejection of an acceptable offer is not observable to future buyers and, therefore, does not alter their offer behavior. This implies that even if the seller deviates from equilibrium by rejecting $p(t)$, her reservation price does not change. Consequently, we can exploit the seller’s indifference in order to calculate her reservation price.

The equilibrium price dynamics works in the same direction as the belief dynamics described above. If either $q(t) > q_1$ or $q(t) = q^*$, then buyers’ offer strategies do not vary over time. Therefore, $p(t)$ is also time-invariant. If $q(t) > q_1$, then the low-type seller receives $c_H$ at a constant rate of $\lambda$, and thus $p(t) = (rc_L + \lambda c_H)/(r + \lambda)$. If $q(t) = q^*$, then $p(t)$ remains constant at $v_L$.

If $q(t) \in (q^*, q_1)$, then $p(t)$ decreases over time (see the solid lines in Figure 3). This is because $q(t)$ decreases over time, and thus buyers become more reluctant to offer $c_H$. To be formal, suppose $q(t) \in (q_{n+1}, q_n)$ for some $n < n^*$. Then, the low-type seller receives offer $c_H$ at rate $\lambda(1 - \Gamma_{L}(s_n))$ as long as $q(t)$ stays in the interval. The rate decreases to $\lambda(1 - \Gamma_{L}(s_{n+1}))$ once $q(t)$ passes $q_{n+1}$. In this manner, the rate at which the low type receives $c_H$ forms a decreasing step function of time until $q(t)$ hits $q^*$. Once $q(t) = q^*$, the low-type seller’s reservation price remains constant at $v_L$.

If $q(t) < q^*$, then $p(t)$ increases over time (see the dashed lines in Figure 3). Contrary to the previous case, $q(t)$ increases over time, and thus buyers become more willing to offer $c_H$. If $q(t) \in (q_{n+1}, q_n)$ for some $n \geq n^*$, then the rate at which the low-type seller receives offer $c_H$ is equal to $\lambda(1 - \Gamma_{L}(s_n))$. Buyers’ beliefs $q(t)$ increase over time and reach $q_n$ in finite time. From this point, and until $q(t)$ reaches $q_{n-1}$, the rate stays constant at $\lambda(1 - \Gamma_{L}(s_{n-1}))$. Similarly to the
previous case, the rate at which the low type receives offer $c_H$ follows an increasing step function until $q(t)$ arrives at $q^*$.

### 3.3 Equilibrium Construction

We now formally construct the unique equilibrium strategy profile. We first determine the unique stationary belief level $q^*$ and the equilibrium strategies on the stationary path. We then characterize the equilibrium strategies along the convergence path, by explicitly solving for the cutoff beliefs $\overline{q}_1, ..., \overline{q}_N$ as well as the low-type seller’s reservation price schedule $p(t)$.

#### 3.3.1 Unique Stationary Path

A necessary condition for $q(t)$ to stay constant is $p(t) = v_L$. If $p(t) > v_L$, then trade takes place only at $c_H$: $p(t)$ can be accepted only by the low-type seller, but no buyer would be willing to pay more than $v_L$ for a low-quality asset. Therefore, such $p(t)$ must be rejected in equilibrium. In this case, unless buyers offer $c_H$ regardless of their signal, $q(t)$ necessarily decreases over time (see equation (1)). If $p(t) < v_L$, then the low-type seller accepts not only $c_H$, but also $p(t)$ with probability 1: otherwise, the buyer could offer a slightly higher price than $p(t)$, which would increase the low-type seller’s acceptance probability to 1. This means that the low-type seller necessarily trades faster than the high type, and thus $q(t)$ increases over time (see equation (2)).

We use the equilibrium condition $p(t) = v_L$ to determine buyers’ equilibrium offer strategies on the stationary path. Denote by $\rho_L (\rho_H)$ the constant rate at which the low-type (high-type) seller receives offer $c_H$ on the stationary path. The low-type seller’s reservation price is then given by

$$ p(t) = \int_t^\infty [(1 - e^{-rx})c_L + e^{-rx}c_H] d(1 - e^{-\rho_L(x-t)}) = \frac{r c_L + \rho_L c_H}{r + \rho_L}. $$

For $p(t) = v_L$, it must be that

$$ \rho_L = \frac{r(v_L - c_L)}{c_H - v_L}. $$

In other words, the low-type seller’s reservation price $p(t)$ remains equal to $v_L$ if she receives offer $c_H$ at a constant rate of $\rho_L = r(v_L - c_L)/(c_H - v_L)$.

Suppose all buyers employ an identical pure strategy of offering $c_H$ if and only if their signal is strictly above $s_n$. Then, the low-type seller receives offer $c_H$ at rate $\lambda(1 - \Gamma_L(s_n))$. Generically, there is no $n$ such that $\rho_L = \lambda(1 - \Gamma_L(s_n))$. This means that the equilibrium condition for the stationary path, $p(t) = v_L$, typically cannot be satisfied if buyers play a pure strategy. In what follows, we focus on the generic case where $\rho_L \neq \lambda(1 - \Gamma_L(s_n))$ for any $n = 1, ..., N$, relegating the discussion on the non-generic case to Section 3.4.2.
Let \( n^\ast \) be the unique integer such that

\[
\lambda (1 - \Gamma_L(s_{n^\ast})) < \rho_L < \lambda (1 - \Gamma_L(s_{n^\ast - 1})).
\]  

(3)

In other words, \( n^\ast \) is the value such that if buyers’ strategies are to offer \( c_H \) if and only if their signal is weakly above \( s_{n^\ast} \), then the low-type seller’s reservation price exceeds \( v_L \) (the second inequality); while if buyers’ strategies are to offer \( c_H \) if and only if their signal is strictly above \( s_{n^\ast} \), then the low-type seller’s reservation price falls short of \( v_L \) (the first inequality). Assumption \( \Box \) ensures that \( n^\ast \) is well-defined. Given \( n^\ast \), let \( \sigma_B^\ast \in (0, 1) \) be the value that satisfies

\[
\rho_L = \lambda (\gamma_L(s_{n^\ast})\sigma_B^\ast + 1 - \Gamma_L(s_{n^\ast})).
\]  

(4)

By construction, \( p(t) \) is equal to \( v_L \) if all subsequent buyers offer \( c_H \) with probability 1 when their signal is strictly above \( s_{n^\ast} \), with probability \( \sigma_B^\ast \) when their signal is \( s_{n^\ast} \), and with probability 0 when their signal is strictly below \( s_{n^\ast} \).

We now determine \( q^\ast \), using the optimality of buyers’ equilibrium strategies. Consider a buyer who has prior belief \( q^\ast \) and receives signal \( s_{n^\ast} \). By Bayes’ rule, his belief updates to

\[
\frac{q^\ast \gamma_H(s_{n^\ast})}{q^\ast \gamma_H(s_{n^\ast}) + (1 - q^\ast)\gamma_L(s_{n^\ast})}.
\]

At this belief, since \( \sigma_B^\ast \in (0, 1) \), the buyer must be indifferent between offering \( c_H \) and \( p(t) = v_L \). This implies that

\[
q^\ast \gamma_H(s_{n^\ast})(v_H - c_H) + (1 - q^\ast)\gamma_L(s_{n^\ast})(v_L - c_H) = 0 \iff q^\ast = \frac{\gamma_L(s_{n^\ast}) c_H - v_L}{\gamma_H(s_{n^\ast}) v_H - c_H}.
\]  

(5)

It remains to pin down \( \sigma_S^\ast \) (the probability that the low-type seller accepts \( v_L \)). We use the fact that \( q(t) \) is time-invariant if and only if the low type trades at the same rate as the high type. The high type accepts only \( c_H \). Therefore, given buyers’ offer strategies characterized by \( (n^\ast, \sigma_B^\ast) \), her trading rate is equal to \( \rho_H = \lambda (\gamma_H(s_{n^\ast})\sigma_B^\ast + 1 - \Gamma_H(s_{n^\ast})) \). If the low type accepts \( v_L \) with probability \( \sigma_S^\ast \), then her trading rate is equal to

\[
\lambda \left[ \gamma_L(s_{n^\ast})\sigma_B^\ast + 1 - \Gamma_L(s_{n^\ast}) + \left( \Gamma_L^-(s_{n^\ast}) + \gamma_L(s_{n^\ast})(1 - \sigma_B^\ast) \right) \sigma_S^\ast \right].
\]

The value of \( \sigma_S^\ast \) must equate the two rates, that is,

\[
\gamma_H(s_{n^\ast})\sigma_B^\ast + 1 - \Gamma_H(s_{n^\ast}) = \gamma_L(s_{n^\ast})\sigma_B^\ast + 1 - \Gamma_L(s_{n^\ast}) + \left( \Gamma_L^-(s_{n^\ast}) + \gamma_L(s_{n^\ast})(1 - \sigma_B^\ast) \right) \sigma_S^\ast.
\]  

(6)
The solution of $\sigma^*_S$ for equation (6) is well-defined in $(0, 1)$, because, due to first-order stochastic dominance,

$$\gamma_L(s_{n^*})\sigma_B^* + 1 - \Gamma_L(s_{n^*}) < \gamma_H(s_{n^*})\sigma_B^* + 1 - \Gamma_H(s_{n^*}) < 1.$$ 

To summarize, generically, there exists a unique stationary path on which buyers’ beliefs $q(t)$ stay constant. The stationary belief level $q^*$ and agents’ equilibrium strategies on the path, described by $n^*$, $\sigma_B^*$, and $\sigma_S^*$, are uniquely determined by equations (3), (4), (5), and (6).

### 3.3.2 Convergence Paths

The equilibrium strategies along the convergence path are fully determined by the cutoff beliefs \{\overline{q}_N, \ldots, \overline{q}_1\} and the low-type seller’s reservation price schedule $p(t)$. We complete the equilibrium construction by jointly identifying \{\overline{q}_N, \ldots, \overline{q}_1\} and $\tilde{p}(q)$. Note that $p(t)$ can be recovered from $\tilde{p}(q)$ and $q(t)$ (i.e., $p(t) = \tilde{p}(q(t))$).

At each cutoff belief $\overline{q}_n$, a buyer with signal $s_n$ must be indifferent between offering $c_H$ and $\tilde{p}(\overline{q}_n)$. This leads to the following condition:

$$\overline{q}_n \gamma_H(s_n)(v_H - c_H) + (1 - \overline{q}_n)\gamma_L(s_n)(v_L - c_H) = \max\{(1 - \overline{q}_n)(v_L - \tilde{p}(\overline{q}_n)), 0\},$$

which is equivalent to

$$\frac{\overline{q}_n}{1 - \overline{q}_n} = \frac{\gamma_L(s_n) c_H - \min\{v_L, \tilde{p}(\overline{q}_n)\}}{\gamma_H(s_n)(v_L - c_H)}.$$ 

The appearance of $\min\{v_L, \tilde{p}(\overline{q}_n)\}$, instead of $\tilde{p}(\overline{q}_n)$, reflects the fact that if $\tilde{p}(\overline{q}_n) \geq v_L$ then the buyer’s expected payoff from offering $\tilde{p}(\overline{q}_n)$ is equal to 0, either because the offer itself is $v_L$, which is accepted only by the low type, or because it is greater than $v_L$, in which case in equilibrium it is rejected with probability 1.

Let $T(q, q')$ denote the length of time it takes buyers’ beliefs to move from $q$ to $q'$ (i.e., $T(q, q')$ is the value such that if $q(t) = q$, then $q(t + T(q, q')) = q'$). For example, suppose $q, q' \in (\overline{q}_{n+1}, \overline{q}_n)$ for some $n < n^*$. In this case, buyers’ beliefs decrease over time, and thus $T(q, q')$ is well-defined only when $q > q'$. In addition, since the trading rate of the type-$a$ seller is equal to $\lambda(1 - \Gamma_a(s_n))$, $T(q, q')$ is the value that satisfies

$$q' = \frac{qe^{-\lambda(1 - \Gamma_a(s_n))T(q, q')}}{qe^{-\lambda(1 - \Gamma_a(s_n))T(q, q')} + (1 - q)e^{-\lambda(1 - \Gamma_a(s_n))T(q, q')}}.$$ 

If $q$ and $q'$ belong to two different partition elements, for example, $(\overline{q}_{n+1}, \overline{q}_n)$ and $(\overline{q}_{n+2}, \overline{q}_{n+1})$, respectively, then $T(q, q')$ can be obtained by separately calculating $T(q, \overline{q}_{n+1})$ and $T(\overline{q}_{n+1}, q')$ and adding them. The case when $q$ and $q'$ lie below $q^*$ can be similarly derived.

Fix $q \in (\overline{q}_{n+1}, \overline{q}_n)$. If $n < n^*$ then $q(t)$ decreases and reaches $\overline{q}_{n+1}$, while if $n \geq n^*$ then $q(t)$
increases and moves to $\bar{q}_n$. For conciseness, define $\tilde{q}$ so that $\tilde{q} = \bar{q}_{n+1}$ if $n < n^*$, while $\tilde{q} = \bar{q}_n$ if $n \geq n^*$. Using the fact that the low-type seller receives $c_H$ at rate $\lambda(1 - \Gamma_L(s_n))$ until $q(t)$ reaches $\tilde{q}$, $\tilde{p}(q)$ can be expressed as follows:

$$\tilde{p}(q) = \int_0^{T(q,\tilde{q})} [(1 - e^{rx})c_L + e^{rx}c_H] \left(1 - e^{-\lambda(1 - \Gamma_L(s_n))x} + e^{-(r + \lambda(1 - \Gamma_L(s_n))T(q,\tilde{q}))} \tilde{p}(\tilde{q}) \right).$$  \hspace{1cm} (8)

Notice that the expression takes a recursive form among $\tilde{p}(\bar{q}_1), ..., \tilde{p}(\bar{q}_N)$. In particular, given $n^*$ and $\tilde{p}(q^*) = v_L$, it is possible to directly calculate $\tilde{p}(\bar{q}_{n^*-1})$ and $\tilde{p}(\bar{q}_{n^*+1})$. Recursively applying equation (8), all other $\tilde{p}(\bar{q}_n)$'s can also be derived.

We now identify all equilibrium variables by combining the two conditions (7) and (8). First, consider $\bar{q}_n$ for $n < n^*$. In this case, $\tilde{p}(\bar{q}_n) > v_L$, and thus equation (7) reduces to

$$\frac{\bar{q}_n}{1 - \bar{q}_n} = \frac{\gamma_L(s_n) c_H - v_L}{\gamma_H(s_n) v_H - c_H}.$$  

Given $\bar{q}_1, ..., \bar{q}_{n^*}$, it is straightforward to calculate $\tilde{p}(q)$ for any $q \geq q^*$ with equation (8). For example, if $q \in (\bar{q}_{n^*}, \bar{q}_{n^*-1}]$, then

$$\tilde{p}(q) = \left(1 - e^{-(r + \lambda(1 - \Gamma_L(s_{n^*}))T(q,q^*))} \right) \frac{r c_L + \lambda(1 - \Gamma_L(s_{n^*})) c_H}{r + \lambda(1 - \Gamma_L(s_{n^*}))} + e^{-(r + \lambda(1 - \Gamma_L(s_{n^*}))T(q,q^*))} v_L.$$  

The determination of $\bar{q}_n$ for $n > n^*$ is more involved, because those cutoffs cannot be separately identified from $\tilde{p}(\bar{q}_n)$: since $\tilde{p}(\bar{q}_n) < v_L$, equation (7) becomes

$$\frac{\bar{q}_n}{1 - \bar{q}_n} = \frac{\gamma_L(s_n) c_H - \tilde{p}(\bar{q}_n)}{\gamma_H(s_n) v_H - c_H}. $$  \hspace{1cm} (9)

Still, it suffices to combine the two conditions (7) and (8), as follows. Assume $n = n^* + 1$, and suppose we continuously decrease $\bar{q}_n$ from $q^*$. The left-hand side in equation (9) obviously decreases, while the right-hand side strictly increases: notice that in equation (8), $\tilde{p}(q)$ is strictly increasing in $q$. Since the left-hand side is larger (respectively, smaller) than the right-hand side if $\bar{q}_n$ is close to $q^*$ (respectively, 0), there exists a unique value of $\bar{q}_n$ that satisfies equation (9). Given $\bar{q}_n$, all other $\bar{q}_{n+k}$'s can be derived by applying the same procedure recursively. Given the cutoffs, the reservation price $\tilde{p}(q)$ can be calculated by equation (8) for any $q < q^*$.

### 3.4 Discussion

Theorem 1 states the generic equilibrium uniqueness in our model. We conclude this section by briefly discussing the uniqueness result and explaining the equilibrium multiplicity in non-generic
3.4.1 Uniqueness

The equilibrium constructed above is essentially the unique equilibrium of the game: for a generic set of parameter values, there does not exist another equilibrium whose outcome does not coincide with the one constructed above almost everywhere. We relegate a formal technical proof to Appendix B but provide the idea behind the uniqueness here.

The equilibrium uniqueness relies on the following two monotonicity properties of any equilibrium strategy profile: (i) Buyers’ beliefs $q(t)$ evolve monotonically (i.e., $q(t)$ either keeps (weakly) increasing or decreasing). (ii) The low-type seller’s reservation price $p(t)$ also evolves monotonically. Given these two properties, it follows that there exists a partition $\{q_N, \ldots, q_1\}$ that describes buyers’ equilibrium offer strategies. The explicit equilibrium construction above then implies that there cannot exist any other equilibrium. These two properties are fairly intuitive, but non-trivial to establish. In particular, if the low-type seller’s reservation price were to decline over time while buyers’ beliefs increased, buyers might become more reluctant to offer $c_H$ when their beliefs are higher, simply because a lower reservation price makes the option of trading only with the low type more attractive. This, in turn, could be consistent with the declining reservation price of the low-type seller. The main thrust of our uniqueness proof is to rule out this possibility.

3.4.2 Non-generic Cases

In the non-generic case where $\rho_L = \lambda(1 - \Gamma_L(s_{n^*}))$ for some $n^*$, the equilibrium uniqueness fails. This follows from the fact that any $q^* \in [q_{n^*+1}, \overline{q}_{n^*}]$ can be supported as the stationary belief level. In fact, buyers’ beliefs do not even need to converge to a certain level, because they can fluctuate in an arbitrary manner within the interval $[q_{n^*+1}, \overline{q}_{n^*}]$. This arises because, unlike buyers’ offer strategies that are determined by the equilibrium requirement that $p(t) = v_L$, the low-type seller’s acceptance strategy of $v_L$ is indeterminate. For instance, the low-type seller may accept $v_L$ with probability 1 until buyers’ beliefs reach $\overline{q}_{n^*}$ and then with a constant probability so that $q(t)$ stays constant. Or, she may reject $v_L$ with probability 1, until buyers’ beliefs hit $\overline{q}_{n^*}$. Buyers’ beliefs may even keep oscillating between $\overline{q}_{n^*}$ and $\overline{q}_{n^*+1}$ (or between any other pair of beliefs in the interval).

Nevertheless, all these equilibria have crucial properties in common. First, within the range $[\overline{q}_{n^*+1}, \overline{q}_{n^*}]$, the low-type seller’s reservation price is necessarily equal to $v_L$. Second, outside $[\overline{q}_{n^*+1}, \overline{q}_{n^*}]$, buyers’ beliefs gradually converge to the interval $[\overline{q}_{n^*+1}, \overline{q}_{n^*}]$, just as they converge to $q^*$ in the generic case. Furthermore, the unique convergence path can be fully characterized as in the generic case: if $q(t) \in [\overline{q}_{n+1}, \overline{q}_n]$ for some $n < n^*$, then the low type trades at rate
Figure 4: Equilibrium belief cutoffs as $\lambda$ varies. For each $n = 1, \ldots, N - 1$, $\lambda^*_n$ is the value such that $\lambda^*_n(1 - \Gamma_L(s_n)) = \rho_L$, and $\lambda^*_0 = \rho_L$ (see Section 3.3.1).

$\lambda(1 - \Gamma_L(s_n))$, while the high type trades at rate $\lambda(1 - \Gamma_H(s_n))$. If $n \geq n^* + 1$, then the low type trades at rate $\lambda$, while the high type trades at rate $\lambda(1 - \Gamma_H(s_n))$. Finally, given the first two properties, it follows that for any initial belief $\hat{q}_i$, all the equilibria are payoff-equivalent. The only difference among the equilibria is the low-type seller’s trading rates while buyers’ beliefs lie in the interval $[q^*_n, q^*_{n+1}]$, as they also depend on the low-type seller’s acceptance strategy.

4 The Role of Search Frictions

In this section, we study how search frictions influence trading dynamics in our model. We illustrate how the equilibrium structure depends on the arrival rate of buyers $\lambda$ and analyze the effects of reducing search frictions on welfare and efficiency. In particular, we study how the low-type seller’s expected payoff and the (random) time to trade for each seller type depend on $\lambda$.

Equilibrium structure. Figure 4 illustrates how the equilibrium structure varies with respect to the arrival rate of buyers $\lambda$. Each equilibrium cutoff belief $\overline{q}_n$ continuously decreases as $\lambda$ increases and stays constant once $\lambda$ reaches the point $(\lambda^*_n - 1)$ at which $\overline{q}_n$ becomes the stationary belief level. The former ($\overline{q}_n$ declining in $\lambda$) corresponds to the region where $\overline{q}_n$ lies in the pessimistic region (i.e., $\overline{q}_n < q^*$), while the latter ($\overline{q}_n$ being independent of $\lambda$) occurs when $\overline{q}_n$ belongs to the weakly

---

8See the proof of Proposition 1 in Appendix B for a formal proof. In addition, see Appendix A for the equilibrium structure when search frictions are so large that Assumption 1 is violated (i.e., $\lambda < \lambda^*_0$ in Figure 4) and the online appendix for the equilibrium outcome in the search-frictionless limit (i.e., as $\lambda$ tends to infinity).
optimistic region (i.e., \( \bar{q}_n \geq q^* \)). The thicker lines indicate the stationary belief \( q^* \) for each \( \lambda \). As illustrated in the previous section, \( q^* \) is necessarily equal to one of \( \bar{q}_n \)'s and takes a downward jump at each non-generic point \( \lambda^{n\ast}_n \). Intuitively, a decrease in search frictions increases the low-type seller’s reservation price by reducing the cost of waiting. This makes buyers less willing to offer \( p(t) \) and, therefore, more willing to offer \( c_H \). When \( \lambda < \lambda^{n\ast}_{n-1} \), \( p(t) < v_L \), and thus this effect is strict and \( \bar{q}_n \) strictly decreases. When \( \lambda > \lambda^{n\ast}_{n-1} \), \( p(t) \geq v_L \), and thus buyers’ incentives to offer \( p(t) \) are independent of the actual value of \( p(t) \). This makes \( \bar{q}_n \) also independent of \( \lambda \).

**Efficiency and welfare.** An increase in \( \lambda \) would have obvious effects on efficiency and welfare if agents were to not adjust their strategies in response. However, agents do adjust their strategies. In particular, for a fixed length of time, the average number of arriving buyers is increasing in \( \lambda \). This means that the seller’s time-on-the-market carries more information and, therefore, buyers’ beliefs evolve faster as \( \lambda \) increases.

For the case of optimistic priors (i.e., \( \hat{q} > q^* \)), this implies that buyers apply higher cutoff signals earlier in time. This counters the direct effect of an increase in \( \lambda \) and potentially slows down trade. A crucial observation is that, since \( \lambda \) affects the trading rates of both types proportionately, the probability of trade while buyers’ beliefs travel, for example, from \( \bar{q}_n \) to \( \bar{q}_{n+1} \) is independent of \( \lambda \). When \( \lambda \) increases, \( T(\bar{q}_n, \bar{q}_{n+1}) \) proportionally decreases, so that \( \lambda T(\bar{q}_n, \bar{q}_{n+1}) \) remains constant. Figure 5 visualizes this effect (the left panel) and also illustrates the aggregate effect on the low-type seller’s time to trade (the right panel). The total probability that each seller type receives
$c_H$ for $T(q_n, q_{n+1})$ is constant, but each type receives $c_H$ more intensively at earlier times as $\lambda$ increases. Therefore, an increase in $\lambda$ necessarily increases the low-type seller’s expected payoff and reduces both seller types’ times to trade in the sense of first-order stochastic dominance.

For the case of pessimistic priors (i.e., $\hat{q} \leq q^*$), faster evolution of buyers’ beliefs implies that buyers apply lower cutoff signals earlier in time. This clearly reduces the high-type seller’s time to trade, as she trades only at $c_H$, and increases the low-type seller’s expected payoff, as it depends only on the rate at which she receives offer $c_H$. Its effect on the low-type seller’s time to trade, however, is ambiguous. Consider a small increase of $\lambda$ around $\lambda_n^*$, which makes the stationary belief level $q^*$ jump down from $q_n$ to $q_{n+1}$, and suppose $\hat{q}$ is close to $q_{n+1}$. Before the change, the low-type seller trades at rate $\lambda$ until $q(t)$ reaches $q_n$ (i.e., for $T(q_n, q_n)$ length of time) and at rate $\rho_H$ thereafter. After the change, her trading rate quickly becomes equal to $\rho_H$. Since $\rho_H$ is continuous in $\lambda$ (see Lemma 4 in the appendix), the change clearly slows down trade of the low type. Intuitively, this is because an increase in $\lambda$ increases the low-type seller’s incentive to reject $p(t)$ and wait for $c_H$. Note that this indirect negative effect operates only for the low type, because the high type accepts only $c_H$. This explains why reducing search frictions affects the two seller types differently.

Proposition 1 formalizes the ongoing discussion. For the low-type seller’s time to trade in the pessimistic prior case, it provides sufficient conditions under which the indirect effect in the previous paragraph is not so strong that the overall effect is still positive. Importantly, the affirmative result holds whenever search frictions are sufficiently small.

**Proposition 1** Fix $\hat{q} < q_1$, and let $\tau_a$ denote the random time at which the type-$a$ seller trades.

- The low-type seller’s expected payoff at time $0$, $\tilde{p}(\hat{q})$, increases in $\lambda$.
- The high-type seller’s time to trade $\tau_H$ decreases in $\lambda$ in the sense of first-order stochastic dominance.
- The low-type seller’s time to trade $\tau_L$ decreases in $\lambda$ in the sense of first-order stochastic dominance, provided that $\hat{q} > q^*$ or $\lambda > \lambda_{N-1}^*$.

**Proof.** See Appendix B.

## 5 Informativeness of Buyers’ Signals

In this section, we explore the effects of varying the informativeness of buyers’ signals. In particular, we address whether an increase in the informativeness, which presumably helps reduce information asymmetry in the market, can be beneficial to the seller and improve market efficiency. This exercise provides implications for policies that aim to enhance transparency in markets.
Specifically, we study the effects on the equilibrium structure, market efficiency, and welfare of varying the inspection technology in the sense of Blackwell (1951). For tractability, as well as to focus on purely informational aspects, we restrict attention to the parameter space in which the equilibrium cutoff belief associated with the highest signal $s_N$, $\overline{q}_N$, serves as the stationary cutoff belief $q^*$ (i.e., $n^* = N$).

**Blackwell informativeness.** An inspection technology is described by a tuple $\Gamma = (S, \gamma_L, \gamma_H)$, where $S$ denotes the set of signals and $\gamma_a$ represents the signal generating process conditional on type $a = L, H$. According to Blackwell (1951), $\Gamma' = (S', \gamma'_L, \gamma'_H)$ is less informative than $\Gamma = (S, \gamma_L, \gamma_H)$ if there exists a (Markov) matrix $M = (m_{ij})_{N' \times N}$ such that all elements are non-negative, each row sums to 1 (i.e., $\sum_j m_{ij} = 1$ for each $i$), and for each $a = L, H$ and $i = 1, \ldots, N'$:

$$\gamma'_a(s'_i) = \sum_j m_{ij} \gamma_a(s_j),$$

where $N$ (respectively, $N'$) represents the cardinality of the set $S$ (respectively, $S'$). Intuitively, $\Gamma'$ can be obtained by garbling $\Gamma$ with noises ($M$) and, therefore, is less informative than $\Gamma$.

The following lemma provides a general implication of Blackwell informativeness for the equilibrium structure in our model.

**Lemma 1** If $\Gamma$ is more informative than $\Gamma'$, then the maximal equilibrium cutoff belief ($\overline{q}_1$) is higher, while the minimal equilibrium cutoff belief ($\overline{q}_N$) is lower, under $\Gamma$ than under $\Gamma'$.

**Proof.** See Appendix B.

In other words, a decrease in the informativeness of buyers’ signals decreases the maximal equilibrium cutoff belief $\overline{q}_1$ (above which buyers always offer $c_H$), while increases the minimal equilibrium cutoff belief $\overline{q}_N$ (below which buyers never offer $c_H$). For the intuition, notice that a decrease in the informativeness of buyers’ signals implies that $s_1$ becomes a weaker signal of the low quality, while $s_N$ becomes a weaker signal of the high quality. Therefore, buyers’ incentives to offer $c_H$ increase conditional on $s_1$, while decrease conditional on $s_N$. The result follows by combining this with the fact that $\overline{q}_n$ is the point at which the buyer obtains zero expected payoff if he offers $c_H$ with signal $s_n$.

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9The necessary and sufficient condition for this is $\lambda_\gamma_L(s_N)(c_H - v_L) > r(v_L - c_L)$. For a given set of parameters, this simply puts a lower bound on $\gamma_L(s_N)$, ruling out the possibility that the highest signal $s_N$ effectively reveals the high quality of the asset. Notice that this condition necessarily holds if the seller is sufficiently patient (i.e., $r$ is close to 0) or search frictions are sufficiently small (i.e., $\lambda$ is sufficiently large), provided that $\gamma_L(s_N) > 0$.

10To put it differently, define an $N \times 2$ matrix $\Gamma$ whose columns correspond to $\gamma_L$ and $\gamma_H$ and, similarly, an $N' \times 2$ matrix $\Gamma'$ for $(S', \gamma'_L, \gamma'_H)$. $\Gamma'$ is less informative than $\Gamma$ if there exists a Markov matrix such that $\Gamma' = M \cdot \Gamma$.  

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Importantly, Lemma 1 is the only general implication of Blackwell informativeness. In other words, there is no general restriction on the behavior of other equilibrium cutoff beliefs, \( \bar{q}_2, \ldots, \bar{q}_{N-1} \): each of them may increase or decrease depending on the specific form of garbling. In fact, even the number of signals is indeterminate: \( N' \) may be larger or smaller than \( N \). This is because the generality of Blackwell informativeness also makes the criterion too permissive (see, e.g., Ponssard, 1975).

Equilibrium in the uninformative limit. Since \( \bar{q}_1 > \bar{q}_n > \bar{q}_N \) for any \( n = 2, \ldots, N - 1 \), Lemma 1 implies that dispersion among \( \bar{q}_n \)'s reduces as the informativeness of buyers’ signals decreases. In the limit, dispersion completely disappears, and thus \( \bar{q}_n = (c_H - v_L)/(v_H - v_L) \) for all \( n \) (see equation (7)). This means that in the uninformative limit, there effectively exists only one equilibrium cutoff belief \( (c_H - v_L)/(v_H - v_L) \), above which all buyers offer \( c_H \) (therefore, buyers’ beliefs stay constant) and below which buyers offer only \( p(t) \) (therefore, buyers’ beliefs strictly increase). Although this is straightforward from the fact that the uninformative limit simply corresponds to the case without buyer signals, it demonstrates, yet another way, that the decreasing trading dynamics, which is the main novelty of this paper, stems from informative private buyer signals.

Efficiency and welfare. The following proposition illustrates the effects of changing the informativeness of buyers’ signals on efficiency and welfare. It demonstrates that the effects are in general ambiguous: a decrease in the informativeness of buyers’ signals tends to decrease the low-type seller’s expected payoff and slow down trade if the initial prior belief \( \hat{q} \) is relatively low, while it may have the opposite effects if the initial prior belief \( \hat{q} \) is relatively high.

**Proposition 2** Fix \( \hat{q} < (c_H - v_L)/(v_H - v_L) \), and consider an inspection technology \( \Gamma' \) such that \( \hat{q} < \bar{q}_N \). If \( \Gamma' \) is less informative than \( \Gamma \), then

- the low-type seller’s expected payoff \( p(\hat{q}) \) is larger under \( \Gamma \) than under \( \Gamma' \),
- the low-type seller’s expected time to trade \( E[\tau_L] \) is smaller under \( \Gamma \) than under \( \Gamma' \), and
- the high-type seller’s time to trade \( \tau_H \) is smaller, in the first-order stochastic dominance sense, under \( \Gamma \) than under \( \Gamma' \).

\(^{11}\)In the online appendix, we introduce a tractable form of garbling technology that preserves the number of signals (uniform garbling) and explain that such a structure permits a stronger, and more intuitive, result about the behavior of the equilibrium structure. Specifically, we show that garbling lowers the equilibrium cutoff belief \( \bar{q}_n \) if and only if the associated likelihood ratio \( \gamma_H(s_n)/\gamma_L(s_n) \) is less than 1. Since the likelihood ratio \( \gamma_H(s_n)/\gamma_L(s_n) \) is strictly increasing in \( n \), this means that there exists \( \tilde{n} \) such that \( \bar{q}_n \) decreases if \( n < \tilde{n} \), while increases if \( n \geq \tilde{n} \).
Now fix $\hat{q} > (c_H - v_L)/(v_H - v_L)$. If $\Gamma$ is sufficiently uninformative, then $\bar{q}_1 < \hat{q}$, and thus all buyers offer $c_H$ with probability 1. Consequently,

- the low-type seller’s expected payoff achieves the maximal value $(r c_L + \lambda c_H)/(r + \lambda)$, and
- each seller type’s time to trade is minimized in the sense of first-order stochastic dominance.

**Proof.** See Appendix B. ■

To understand the first set of results, notice that when the informativeness of buyers’ signals decreases, the highest signal $s_N$ becomes a less convincing indication of the high quality and, therefore, buyers become more reluctant to offer $c_H$ conditional on receiving this signal. This results in lowering the probability that buyers offer $c_H$ on the stationary path as well as increasing the stationary belief level $q^*$ itself. This explains why a decrease in the informativeness lowers the low-type seller’s expected payoff and delays trade of the high type when $\hat{q}$ is relatively low. Its effect on the low-type seller’s time to trade is in general ambiguous (i.e., cannot be ranked in terms of first-order stochastic dominance), because the low-type seller trades at the maximal rate $\lambda$ on the convergence path and a decrease in informativeness delays the convergence to the stationary path. Still, as formally proved in Appendix B, this positive effect cannot be sufficiently large, and thus the low-type seller’s expected time to trade increases as buyers’ signals become less informative.

For the second set of results, notice that as the informativeness of buyers’ signals decreases, from the perspective of a given buyer, other buyers’ signals become less informative as well, and thus the evolution of buyers’ beliefs slows down. This effect is particularly prominent when $\hat{q} > q^*$, because in this case, $q(t)$ reflects only the information received by previous buyers (while if $\hat{q} < q^*$, then $q(t)$ reflects the seller’s acceptance behavior as well). This means that buyers assign higher prior probabilities to the seller being the high type and, therefore, have a stronger incentive to offer $c_H$. Contrary to the previous case where $\hat{q} < q^*$, the fact that each buyer’s own signal is not so informative further increases his incentive to offer $c_H$, because it now means that the lowest signal $s_1$ becomes a weaker indication of the low quality. When the inspection technology is sufficiently uninformative, this effect becomes completely dominant. Buyers’ beliefs $q(t)$ stay constant at $\hat{q}$, and all buyers offer $c_H$ regardless of their signal.

### 6 Applications

In this section, we explain how our model can be extended to address various applied questions of interest. We keep our discussion brief and informal in the main text, relegating all details and formalities to the online appendix.
6.1 Informationally Heterogeneous Buyers

Market participants are informationally heterogeneous. Some have the ability or resources to collect and interpret information, while others do not. Ceteris paribus, information is valuable and, therefore, the former perform better than the latter. This consequence of informational heterogeneity raises a novel question of how to “level the playing field” among market participants. Our model provides unique perspectives to this general question.

It is easy to incorporate informational heterogeneity into our model. Suppose some buyers receive an informative signal (informed), while the others do not (uninformed). Theorem 1 directly applies to this extended environment, because it is isomorphic to the case in which each buyer receives an uninformative signal (whose likelihood ratio is equal to 1) with a certain probability (equal to the proportion of uninformed buyers). Informed buyers behave as in our main model, while uninformed buyers’ behavior can be described by a single belief cutoff, above which they offer \( c_H \) and below which they offer \( p(t) \). The evolution of buyers’ beliefs can also be easily modified to reflect the presence of uninformed buyers.

Consider the following two policies, one to reduce the informational gap between informed and uninformed buyers (e.g., by controlling the informativeness of informed buyers’ signals) and the other to help uninformed buyers become informed (e.g., by educating them or providing necessary resources). Both policies clearly help uninformed buyers catch up with informed buyers. While the direct effects of these policies are clear, our analysis shows that these policies also have indirect effects on uninformed buyers. The first policy, while reducing informed buyers’ payoffs, can increase uninformed buyers’ payoffs. The second policy, while helping uninformed buyers who successfully turn informed, may hurt those who remain uninformed. These indirect effects originate from the observational-learning component of our model. When informed buyers get more precise signals or there are more informed buyers, buyers learn more from the seller’s time-on-the-market. In particular, when the initial prior is relatively high, buyers’ beliefs decline faster, which negatively affects late or uninformed buyers.

6.2 Mandatory Disclosure of Known Defects

In the real estate market, sellers are obligated to disclose any known defects of their property, such as the existence of termite or malfunctions of major systems or appliances, to potential buyers. The failure of proper disclosure leaves the seller responsible for the defects for several years, and legal disputes between previous and current owners are not uncommon. Our model can be used to evaluate the effects of this mandatory disclosure law on market efficiency and welfare.

We can accommodate this feature within our model by assuming that the lowest signal \( s_1 \) completely reveals the low quality of the asset and is observable to all subsequent buyers (i.e., \( s_1 \)
represents the finding of defects, and once a buyer receives $s_1$, the low quality of the asset becomes publicly known. This modification has a significant impact on trading dynamics. Among others, when there are two signals, buyers’ beliefs, conditional on no trade and no realization of signal $s_1$, never decline over time: $q(t)$ either stays constant or strictly increases. In our main model, buyers’ beliefs can decrease because they cannot tell whether delay is due to the realization of low signals or search frictions (i.e., no arrival of buyers). Under mandatory disclosure, buyers can distinguish between the two events, and thus the decreasing trading dynamics cannot arise.

We also show that mandatory disclosure tends to speed up trade (in particular, when the initial prior is relatively high or search frictions are sufficiently small). This is mainly because of the following two effects. First, it lowers the low-type seller’s reservation price. If signal $s_1$ is realized and, therefore, the low quality becomes publicly known, then the seller has no reason to delay trade and would trade at a low price quickly. This possibility also affects the low-type seller before the realization of signal $s_1$, which facilitates trade even further. Second, conditional on no arrival of signal $s_1$, mandatory disclosure makes buyers more optimistic about the quality of the asset and, therefore, more willing to offer $c_H$. This helps both seller types trade faster.

### 6.3 Inspection by Choice

In our main model, inspection is automatic: every buyer receives a signal about the quality of the asset. In various markets, however, inspection is a strategic choice by agents. A seller may simply refuse it or restrict the amount of information available to buyers. Even if inspection is the right of buyers, it is often costly, and thus buyers may choose not to do it. We now explain how to incorporate each of these into our model and what effects they have on equilibrium outcomes.

First, consider the case in which the seller can decide whether to allow each buyer to inspect the asset or not. In this case, the inspection decision itself becomes the seller’s signaling device. Combined with the fact that the high-type seller does not obtain a positive net payoff, this leads to equilibrium multiplicity. It is easy to see that the unique equilibrium outcome of our main model can be supported as an equilibrium, for example, with buyers assigning probability 1 to the low type whenever inspection is denied. Similarly, there is an equilibrium in which the seller always refuses inspection and, therefore, the outcome is equivalent to that of the model without inspection. Of interest is an equilibrium between these two extremes: both seller types allow inspection if and only if buyers’ beliefs are below a certain threshold. This equilibrium always gives the (low-type) seller a weakly higher expected payoff than the previous two equilibria.

Now suppose each buyer decides whether to conduct an inspection at cost $c > 0$. The equilibrium structure and trading dynamics are analogous to those of our main model. Restricting attention to the binary-signal case, buyers inspect the asset and condition their offers on the real-
ized signal if and only if their beliefs are neither sufficiently large nor sufficiently small. Trade always occurs at $c_H$ if buyers’ beliefs are sufficiently high (above $\bar{q}_1$), while only the low-type seller trades if buyers’ beliefs are sufficiently low (below $\bar{q}_2$). In the intermediate range, buyers offer $c_H$ only after signal $s_2$, and thus the high type receives $c_H$ at a higher rate than the low type. Whether buyers’ beliefs increase or decrease on $(\bar{q}_2, \bar{q}_1)$ depends on whether $p(t) > v_L$ or not, which in turn depends on $\lambda$.

We now discuss the effects of reducing the inspection cost $c$. It, naturally, induces more buyers to inspect the asset: $\bar{q}_2$ decreases, while $\bar{q}_1$ increases, so that the region $(\bar{q}_2, \bar{q}_1)$ necessarily expands. Interestingly, its effects on market efficiency and welfare are ambiguous. When the initial prior belief is rather high (so buyers would offer $c_H$ without inspection), a reduction in the inspection cost induces buyers to inspect and offer $c_H$ only with good signals, thereby delaying trade and lowering the seller’s expected payoff. To the contrary, if the initial prior belief is rather low (so buyers would offer only $p(t)$ without inspection), a decrease in the inspection cost helps the low-type seller and speeds up trade of the high type, as it now means that buyers offer $c_H$ at least with good signals. The low-type seller’s time to trade may increase because the low-type seller becomes more reluctant to accept a low price.

7 Robustness

Our model is parsimonious in various dimensions. This allows us to deliver our main insights in a particularly simple fashion as well as analyze the effects of key policy variables. It, however, also raises the question on the robustness of our findings. In this section, we consider three alternative environments, each of which differs from our baseline environment in terms of the cardinality of seller types, the bargaining protocol, or the market structure. We show that our main insights continue to hold in all three environments. We briefly discuss our exercises and discuss the main lessons, while relegating all formalities to the online appendix. We note that all the results in this section are for the case where search frictions are sufficiently small (i.e., $\lambda$ is sufficiently large).

7.1 More Than Two Types

When there are more than two seller types, the analysis becomes significantly more complicated mainly for two reasons. First, buyers’ beliefs can be described only by a multi-dimensional vector, not by a single variable as in the two-type case. The evolution of this belief vector is determined by the pairwise relative rates of trading for different types of assets and, therefore, quite complicated to keep track of. Second, the reservation price of each seller type, except for that of the highest type, changes over time and may even exhibit non-monotonicity. Consequently, buyers’ optimal offer...
strategies become a lot harder to characterize. For instance, buyers’ offer strategies may exhibit a form of non-convexity: after some histories, a buyer may be willing to offer the reservation prices of two distant types, but not those in between.

Nevertheless, the insights from the two-type case are likely to extend to a more general environment. We demonstrate this by considering the case of three types, where the asset can be either of low quality (L), of middle quality (M), or of high quality (H). We illustrate the main characterization result for the three-type case with Figure 6, which represents buyers’ beliefs with a two-dimensional simplex and describes how they evolve starting from any initial point. For each \( a = L, M, H \), we denote by \( q_a(t) \) the probability that the seller is of type \( a \) and by \( p_a(t) \) the reservation price of the type-\( a \) seller. In addition, we let \( q(t) \equiv (q_L(t), q_M(t), q_H(t)) \).

Specifically, we show that, under some conditions, there exists a belief vector \( q^* = (q^*_L, q^*_M, q^*_H) \) such that for any initial belief there exists an equilibrium in which \( q(t) \) converges to \( q^* \), except for the rather obvious case where \( q_H(0) \) is so large that it is an equilibrium that all buyers offer \( c_H \).

The convergence occurs as described with arrows in Figure 6. For instance, suppose that \( q(0) \) lies

---

Figure 6: Evolution of buyers’ beliefs and buyers’ equilibrium offer strategies with three types.
in Area A. Then, buyers offer only the low-type seller’s reservation price \( p_L(t) \) for a while. This makes \( q(t) \) move downward in the simplex. Once \( q(t) \) reaches Path 1 or Path 3, it moves along the path toward \( q^* \), which is accomplished by buyers’ offering the reservation prices of two relevant types \( (p_L(t) \) and \( p_H(t) \) on Path 1, while \( p_L(t) \) and \( p_M(t) \) on Path 3). Once \( q(t) \) arrives at \( q^* \), buyers offer the reservation prices of all three types with positive probabilities and their beliefs do not change thereafter.

The result suggests that the central lessons from our model go beyond the simple two-type case. Depending on the initial belief, the reputation of an asset can evolve in various different directions. With more than two types, an asset’s reputation cannot be measured by a single variable any longer. Still, there is a sense in which reputation evolves in a monotone way: the probability of each type tends to increase (decrease) over time if it is relatively large (small) and eventually converges to a certain point.

Furthermore, the underlying economic forces are, reassuringly, similar to those in the two-type case. If buyers initially assign a large probability to the low type (Area A), then they offer only \( p_L(t) \), which is accepted only by the low type. Therefore, delay is mainly attributed to higher types’ resistance to accept a low price, and thus the reputation of the asset improves over time. If the initial probability of the high type is relatively large (Area C), then buyers offer \( p_H(t) \), unless they observe a particularly bad signal. Therefore, delay mainly conveys negative information about the quality of the asset, and thus the reputation deteriorates over time. When the initial probability of the middle type is large (Area B), delay is interpreted as a mixture of these two effects. On the one hand, it indicates the high type’s unwillingness to trade at a mediocre price \( p_M(t) \), thereby increasing the probability of the high type. On the other hand, it also suggests the possibility that all previous buyers have received sufficiently bad signals about the quality of the asset, thereby increasing the relative probability of the low type as well. To the extent to which these economic forces are sensible, our results are likely to carry over to a more general environment, although exponentially increasing technical difficulties do not allow us to formally obtain those results with more types.

### 7.2 Alternative Bargaining Protocols

We now demonstrate that our main insights do not depend on the specific bargaining protocol we consider. Although our main bargaining protocol - that uninformed buyers make price offers to the informed seller - is the most widely adopted one in the literature, it also exhibits some properties that might be considered undesirable or implausible. In particular, the high(est) seller type never obtains a strictly positive (net) expected payoff and, therefore, does not play an active role in the model. We demonstrate that the central lessons from our model are not subject to this particular...
property by considering other bargaining protocols.

Specifically, we consider the following three bargaining protocols:\footnote{The first bargaining protocol was developed by Wolinsky (1990) and has been adopted by several subsequent studies (see, e.g., Blouin and Serrano, 2001; Blouin, 2003). The second one is due to Compte and Jehiel (2010) and adopted by Lauermann and Wolinsky (2015) in a similar context to ours. For the adoption of the last one in the context of dynamic adverse selection, see, for example, Lauermann and Wolinsky (2011); Gerardi, Hörner and Maestri (2014); Palazzo (2015).}

1. Simultaneous announcement game: when the seller meets a buyer, the following normal form game is played: the buyer and the seller simultaneously choose whether to play $T$ (tough) or $S$ (soft). Their choices lead to the trading outcomes summarized in the following table:

<table>
<thead>
<tr>
<th></th>
<th>Buyer</th>
<th>Seller</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$</td>
<td>Trade at $p_M$</td>
<td>Trade at $p_L$</td>
</tr>
<tr>
<td>$T$</td>
<td>Trade at $p_H$</td>
<td>No trade</td>
</tr>
</tbody>
</table>

The prices $p_H \in (c_H, v_H)$, $p_M \in (v_L, c_H)$ and $p_L \in (c_L, v_L)$ are exogenously given and fixed across different meetings.

2. Random proposals bargaining: in each meeting, nature draws a price from an exogenously given distribution function and proposes the price to both parties. Both the seller and the buyer can decide only whether to accept the price or not. For tractability, we consider a simplified version with only two prices, $p_L \in (c_L, v_L)$ and $p_H \in (c_H, v_H)$.

3. Price offers by the seller: the seller, who is the informed player, makes price offers to buyers (uninformed players).

Importantly, all these bargaining protocols yield a strictly positive expected payoff to the high type: with the first two bargaining protocols, the high-type seller eventually trades at $p_H > c_H$. With the last one, due to the signaling nature of the game, there exist many equilibria. In some equilibria, the high-type seller still obtains zero (net) expected payoff, either through no trade at all or trade only at $c_H$. However, there also exist equilibria in which the high-type seller trades at a higher price than $c_H$ and obtains a strictly positive expected payoff.

Despite this difference from our main model and various differences among themselves, all three specifications yield an equilibrium structure and trading dynamics similar to those of our main model. Specifically, there exists an equilibrium (or a class of equilibria) which behaves as in...
Theorem 1: there exists a stationary belief level \( q^* \) such that buyers’ beliefs decrease if \( q(t) > q^* \), increase if \( q(t) < q^* \), and stay constant if \( q(t) = q^* \). Buyers’ optimal strategies are described by a finite partition \( \{ \bar{q}_{N+1} = 0, \bar{q}_N, ..., \bar{q}_0 = 1 \} \). The underlying logic for all three bargaining protocols is identical to that of our main model. When buyers assign a relatively large probability to the seller being the high type (above \( q^* \)), they are willing to trade at a high price unless they receive a sufficiently unfavorable signal. This induces the low-type seller to insist on a high price. Since the high type generates favorable signals more often than the low type, buyers’ beliefs decrease conditional on no trade. In the opposite case when buyers assign a small probability to the high type, they rarely agree on a high price. This provides an incentive for the low type to trade early at a reasonable price. Since the high type, due to her higher cost, still insists on a high price, delay is interpreted as good news and buyers’ beliefs increase over time. When buyers’ beliefs are equal to \( q^* \), these opposing effects are balanced and, therefore, buyers’ beliefs stay constant.

7.3 Alternative Market Structure

In our model, the seller faces at most a single buyer at each time (bilateral meeting). Together with the bargaining protocol, this implies that each buyer has temporary monopsony power, which eventually leads to the Diamond paradox outcome (see footnote [5]). We now introduce instantaneous competition among buyers and illustrate that our main insights continue to hold under a competitive market structure.

We consider the case where multiple buyers arrive simultaneously and offer prices competitively. For tractability, we assume that all buyers in the same cohort observe a common signal about the quality of the asset. This allows us to take a reduced-form approach about buyers’ offer strategies, as in Daley and Green (2012), Fuchs and Skrzypacz (2015), and Fuchs, Öry and Skrzypacz (2014): their behavior can be summarized by the zero-profit condition and an appropriate consistency condition, without explicitly solving for the bidding equilibrium among buyers. The validity of this approach is straightforward to verify by applying the usual Bertrand competition logic.

This alternative market structure exhibits several different features from our baseline model. The zero-profit condition implies that trade occurs either at \( v_L \) (accepted only by the low type) or at the following expected value of the asset (accepted by both types):

\[
p_n(q(t)) = \frac{q(t)\gamma_H(s_n)v_H + (1 - q(t))\gamma_L(s_n)v_L}{q(t)\gamma_H(s_n) + (1 - q(t))\gamma_L(s_n)}.
\]

This implies that price offers are effectively bounded below by \( v_L \). In addition, the high-type seller may obtain a strictly positive expected payoff: observe that \( p_n(q(t)) \) exceeds \( c_H \) if \( q(t) \) is sufficiently large. Finally, prices acceptable to the high-type seller are not unique at each point in
time, and the high-type seller’s reservation price also varies over time.

Nevertheless, our main insights continue to apply to this alternative model. Figure 7 depicts the equilibrium trading dynamics. Unlike in our baseline model, the stationary belief path is not unique: any belief level in the interval $[q^*, \overline{q}]$ can give rise to a stationary path.\footnote{This equilibrium multiplicity with competitive buyers has been observed in the literature before. See, for example, Daley and Green (2012) and Fuchs and Skrzypacz (2015).} While the low-type seller’s reservation price must be equal to $v_L$ on any stationary path, the high-type seller’s reservation price varies across different stationary paths. Fixing a stationary belief level $q^* \in [q^*, \overline{q}]$, however, the model generates similar trading patterns to those of our baseline model. An equilibrium still can be described by a finite partition $\{q_{N+1} = 0, q_N, ..., q_0 = 1\}$: if $q(t) \in (q_{n+1}, q_n)$, then buyers offer a price acceptable to both types if and only if they receive a signal above $s_n$ (although the price depends both on the realized signal and the time). Buyers’ beliefs $q(t)$ decrease over time if $q(t) > q^*$, while increase if $q(t) < q^*$. The economic forces behind these patterns are also identical to those for our baseline model.

8 Conclusion

We conclude by discussing various empirical implications and potential directions for future research.
8.1 Empirical Implications

Our model environment is stylized, abstracting away from many important details in real markets. As always, a certain degree of abstraction is unavoidable to obtain clean and fundamental economic insights. On the other hand, taking such a model to data requires additional steps to account for various factors that are not present in the model. For instance, brokers and list prices play an important role in the real estate market (see, e.g., Horowitz, 1992; Merlo and Ortalo-Magne, 2004; Hendel, Nevo and Ortalo-Magne, 2009). Unemployment durations are also affected by other factors, such as skill depreciation and worker discouragement (see, e.g., Pissarides, 1992; Gonzalez and Shi, 2010). Nevertheless, our model generates some novel and robust predictions regarding market outcomes, some of which are potentially testable. It is beyond the scope of this paper to develop a complete empirical strategy. We provide a list of potentially testable predictions of our model and discuss each of them briefly.

As shown in Sections 3 and 7, buyers’ beliefs and (reservation) prices always move in the same direction. In what follows, we say that the equilibrium trading dynamics exhibits the increasing (decreasing) pattern if they increase (decrease) over time.

Prediction 1 The trading dynamics exhibits the decreasing pattern if an asset’s initial reputation is high and the increasing pattern if the initial reputation is low.

This prediction restates our main result. The simplicity of this relationship between an initial reputation and trading dynamics is desirable for empirical purposes. One potential obstacle lies in the difficulty of measuring (initial) reputations. Although this is a non-trivial task in itself, there typically exist observable characteristics that are related to an asset’s (seller’s) reputation and, therefore, can be used to construct a reputation variable. For instance, the neighborhood, vintage, building company, and owner history provide information about a property’s quality. Similarly, a worker’s education and prior employment histories would affect his reputation in the labor market.

Our next prediction links the pattern of trading dynamics to the evolution of trading probability (equivalently, volume) over time.

Prediction 2 If the trading dynamics exhibits the decreasing (increasing) pattern, then the overall trading probability also decreases (increases) initially.

The result follows from the fact that the frequency with which buyers offer $c_H$ is an increasing function of their beliefs. If $\hat{q} > q^*$, then trade occurs only at $c_H$ until $q(t)$ reaches $q^*$. This immediately implies that trade occurs less frequently over time. If $\hat{q} < q^*$, then the low-type seller trades at a constant rate of $\lambda$ until $q(t)$ reaches $q^*$, while the trading rate of the high-type seller increases. This co-movement of trading pattern and trading probability does not extend into the full time horizon: it is valid along the convergence path, but not at the moment of convergence.
(i.e., when $q(t)$ reaches $q^*$). The trading rate of the high-type seller is always monotone over time. However, when $\hat{q} < q^*$ (i.e., the increasing pattern), the trading rate of the low type jumps down from $\lambda$ to $\rho_H$ at the end of the convergence path. If $\hat{q} > q^*$ (i.e., the decreasing pattern), then the trading rate changes from $1 - \Gamma_L(s_{n^*} - 1)$ to $\rho_H = 1 - \Gamma_H(s_{n^*}) + \gamma_H(s_{n^*})\sigma_B$. Depending on the parameter values, the latter can exceed the former.

We now relate the pattern of trading dynamics to three market characteristics.

**Prediction 3** *The decreasing pattern is more likely to arise with little gains from trade of low quality (i.e., relatively small $v_L - c_L$), small search frictions (i.e., large $\lambda$), and a good inspection technology (i.e., informative buyer signals).*

The first observation is based on the discussion in Appendix A, while the latter two follow from Sections 4 and 5. These results could be useful in interpreting both cross-sectional data and time series. For example, if both the search technology and the inspection technology have improved over time, then the decreasing pattern is more likely to arise in recent data than in old data.

Our final prediction is concerned with the relationship between trading dynamics and the nature of inspection. Specifically, we compare the case when inspection is mainly about finding a fatal flaw (red flag) and the case when inspection may reveal a particular merit of the asset (green flag). Formally, we assume that there are only two signals, $s_1$ and $s_2$, and compare the following two cases: for $\gamma, \epsilon > 0$,

<table>
<thead>
<tr>
<th>Red flag</th>
<th>Green flag</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma_L(s_1) = \gamma$, $\gamma_H(s_1) = \epsilon$, $\gamma_L(s_2) = 1 - \gamma$, $\gamma_H(s_2) = 1 - \epsilon$, $\gamma_L(s_2) = \gamma$, $\gamma_H(s_2) = 1 - \gamma$</td>
<td>$\gamma_L(s_1) = 1 - \epsilon$, $\gamma_H(s_1) = 1 - \gamma$, $\gamma_L(s_2) = \epsilon$, $\gamma_H(s_2) = \gamma$.</td>
</tr>
</tbody>
</table>

If $\epsilon$ is sufficiently small, then $s_1$ (red flag) is a sufficiently informative signal about low quality in the case represented by the left-hand column, while $s_2$ (green flag) is a sufficiently good signal about high quality in the case represented by the right-hand column. Assuming that $\gamma$ is not particularly large (precisely, $\lambda(1 - \gamma) > \rho_L$), the following prediction is straightforward to obtain by applying Theorem 1 to each case (see the online appendix for formal arguments).

**Prediction 4** *The decreasing pattern is more likely to arise when the inspection technology is of a red-flag kind than when it is of a green-flag kind.*

### 8.2 Directions for Future Research

Our insights and results suggest several directions for future research. One of our maintained assumptions is that buyers’ offers are private and not observable to future buyers. Hörner and Vieille (2009) show that it is a crucial assumption and the equilibrium dynamics dramatically changes
if buyers’ offers are public. In particular, with public offers, “bargaining impasse” can arise. As also suggested by Horner and Vieille (2009), it could be interesting to allow for buyer inspection in the model with public offers and investigate its impact on bargaining impasse. Our model is a dynamic trading model with random search, but there is a growing literature on directed search with adverse selection (see, e.g., Guerrieri, Shimer and Wright 2010, Guerrieri and Shimer, 2014, Chang, 2014). Introducing buyer inspection into models of directed search might also lead to interesting insights or predictions. Finally, we assume, crucially, that buyers are short-lived. This assumption can be relaxed in various ways. For example, one can consider a market environment in which there are many sellers and buyers and all agents go through sequential search until they trade (see, e.g., Wolinsky, 1990; Blouin and Serrano, 2001; Moreno and Wooders, 2010). Such a model, whether stationary or non-stationary, would embed our model into a market setting and endogenize buyers’ outside options. Another possibility is to introduce and endogenize buyers’ optimal timing decisions (i.e., when to arrive and make an offer to the seller). Clearly, it would influence the informational content of time-on-the-market and, therefore, potentially make buyers’ inference problems even more intriguing.

Appendix A: Further Comments on Theorem 1

In this appendix, we explain how Theorem 1 can be modified to accommodate the following three alternative specifications of the model: (i) the case of large search frictions (i.e., when Assumption 1 is violated); (ii) the case of a continuum of possible inspection outcomes; and (iii) the case of no gains from trade for one of the seller types.

Large Search Frictions

Suppose search frictions are so large that Assumption 1 is violated. In this case, the low-type seller’s reservation price $p(t)$ never exceeds $v_L$. To be formal, suppose that the low-type seller expects to receive offer $c_H$ with probability $\frac{1}{2}$ from the next buyer. Her reservation price is then equal to

$$p^* = \frac{rc_L + \lambda c_H}{r + \lambda}.$$ 

If Assumption 1 is violated (i.e., $r(v_L - c_L) < \lambda(c_H - v_L)$), then $p^* < v_L$. Since $p(t)$ can never exceed $p^*$, it follows that $p(t) < v_L$ for any $t$.

An immediate consequence is that no buyer makes a losing offer and, therefore, the low-type seller always trades. This implies that buyers’ beliefs are always non-decreasing over time: $q(t)$ stays constant if buyers offer $c_H$ regardless of their signal. In all other cases, $q(t)$ strictly increases over time. The equilibrium is again described by a finite partition $\{q_{N+1} = 0, q_N, \ldots, q_0 = 1\}$, and $q_1$ plays the role as the stationary belief: if $q(t) \in (q_{n+1}, q_n)$, then buyers offer $c_H$ only when their signal is above $s_n$. The low-type seller always accepts both $p(t)$ and $c_H$. Consequently, $q(t)$ strictly increases if $q(t) < q_1$, while it stays constant if $q(t) \geq q_1$. These properties are almost identical to
those of the case when \( n^* = 1 \). The only difference is that when \( q(t) = \bar{q}_1 \), buyers always offer \( c_H \) (i.e., no randomization even with signal \( s_1 \)), and the low-type seller’s reservation price is equal to \( p^* \) (instead of \( v_L \)) when Assumption 1 is violated.

### A Continuum of Signals

Although we have considered an environment with a finite number of signals, most equilibrium properties carry over to the case with a continuum of signals. There is a unique belief level \( q^* \) which can support the stationary path: suppose the signal space \( S \) is given by an interval \([s, \bar{s}]\) and \( \Gamma_a(\cdot) \) (respectively, \( \gamma_a(\cdot) \)) now represents the distribution (respectively, density) function of signals conditional on type \( a \). Let \( s^\ast \) be the unique value that satisfies

\[
\lambda(1 - \Gamma_L(s^\ast)) = \rho_L = \frac{r(v_L - c_L)}{c_H - v_L}.
\]

For the stationary path to be sustained, it suffices that each buyer offers \( c_H \) if and only if his signal is above \( s^\ast \). Buyers’ beliefs, whether they are above or below \( q^* \), gradually converge to \( q^* \). The low-type seller’s reservation price also gradually converges to \( v_L \).

A direct characterization of the model with a continuum of signals is involved, because buyers’ cutoff signals would continuously change over time. This particularly complicates the analysis of the case where the initial belief is below \( q^* \), because all equilibrium functions influence one another and must be simultaneously determined. Still, an equilibrium can be constructed by approximating the continuous distribution with discrete distributions and taking the limit of resulting equilibria. The limit equilibrium is characterized by the following system of equations: denote by \( s(t) \) the cutoff signal above which buyers offer \( c_H \). Then,

\[
\frac{q(t)}{1 - q(t)} = \frac{\gamma_L(s(t))}{\gamma_H(s(t))} \frac{c_H - \min\{v_L, p(t)\}}{v_H - c_H},
\]

\[
r(p(t) - c_L) = \lambda(1 - \Gamma_L(s(t)))(c_H - p(t)) + \dot{p}(t),
\]

and

\[
\dot{q}(t) = \begin{cases} 
q(t)(1 - q(t))\lambda(\Gamma_H(s(t)) - \Gamma_L(s(t))), & \text{if } q(t) > q^*, \\
q(t)(1 - q(t))\lambda\Gamma_H(s(t)), & \text{if } q(t) < q^*.
\end{cases}
\]

### No Gap at the Bottom or at the Top

We have assumed that there are positive gains from trade for both seller types (i.e., \( v_a > c_a \) for both \( a = H, L \)). Although this case has been more widely considered in the literature, the cases with no gap at the bottom (\( v_L = c_L \)) or at the top (\( v_H = c_H \)) also have been studied: see, e.g., Taylor (1999); Zhu (2012) for the former case and Fuchs and Skrzypacz (2013); Fuchs, Öry and Skrzypacz (2014) for the latter case. To make comparisons to those papers more transparent, we explain what happens if there is no gap at the bottom or at the top in our model.

If there is no gap at the top (i.e., \( v_H = c_H \)), the result is trivial in our model. Due to the Diamond paradox, all buyers offer only \( c_L \). The low-type seller trades with the first buyer, while the high-type seller never trades. In other words, the equilibrium is essentially identical to that of the complete-information case (i.e., the case when the seller is known to be the low type). We note
that this triviality is driven by our two-type restriction, and the existing papers with the no-gap-at-the-top assumption typically consider a continuous type space.

If there is no gap at the bottom (i.e., \( v_L = c_L \)), then buyers have no incentive to target only the low-type seller and offer \( p(t) \). This means that the problem shrinks only to when buyers have an incentive to offer \( c_H \). The equilibrium is again characterized by a finite partition \( \{ \tau_{N+1} = 0, \tau_N, \ldots, \tau_0 = 1 \} \) such that if \( q(t) \in (\tau_{n+1}, \tau_n) \), then buyers offer \( c_H \) if and only if their signal is strictly above \( s_n \). \( \tau_N \) plays the role as the stationary belief, in that \( q(t) \) strictly decreases if \( q(t) > \tau_N \) and stays constant if \( q(t) = \tau_N \). A crucial difference is that if \( \hat{q} \leq \tau_N \), then buyers never offer \( c_H \) and, therefore, gains from trade are never realized. Intuitively, this is when the probability of the high type is so low that buyers’ expected value of the asset does not exceed \( c_H \) even conditional on the best signal. Since buyers also have no incentive to trade with the low-type seller, there is no scope for trade and the market essentially breaks down. There are equilibria in which the low-type seller trades at \( v_L = c_L \) with a positive probability (insofar as buyers’ beliefs stay below \( \tau_N \)), but \( c_H \) is never offered in any equilibrium.

**Appendix B: Omitted Proofs**

**Proof of Theorem [1]**

We formally establish the uniqueness result. We first establish the monotonicity of buyers’ beliefs. As explained in the main text, if \( p(t) < v_L \) then \( q(t) \) necessarily increases over time, while if \( p(t) > v_L \) then \( q(t) \) decreases. Therefore, the following lemma suffices to prove the belief monotonicity.

**Lemma 2** In any equilibrium, \( q(t) \leq q^* \) if, and only if, \( p(t) \leq v_L \).

**Proof.** We establish the result in three steps.

1. If \( q(t) < q^* \), then \( p(t) < v_L \).

Suppose \( q(t) < q^* \), but \( p(t) > v_L \). Then, there must exist \( t' \in (t, \infty) \) such that \( p(t') = v_L \). To see this, suppose, toward a contradiction, that \( p(t') > v_L \) for any \( t' > t \). This implies that \( q(\cdot) \) keeps decreasing. This, in turn, implies that there must exist \( q_\infty \in [0, q(t)) \) such that \( q(\cdot) \) converges to \( q_\infty \). In the long run, both types must trade at approximately the same rate, which can be the case only when buyers either almost always offer \( c_H \) or almost never offer \( c_H \). Since \( q(t) < q^* \), the former obviously cannot be true. The latter also cannot be the case, because if so, the low-type seller’s reservation price would be close to \( c_L \), which is strictly smaller than \( v_L \). This establishes the claim that there must exist \( t' > t \) such that \( p(t') = v_L \). Let \( t' \) be the smallest value such that \( p(t') = v_L \). Then, for any \( x \in (t, t'), p(x) > v_L \). This implies that \( q(x) \leq q(t) < q^* \) and the probability that each buyer who arrives between \( t \) and \( t' \) offers \( c_H \) to the low-type seller must be strictly smaller than \( \gamma_L(s_n^*) \sigma_B^* + 1 - \Gamma_L(s_n^*) \). Combining this with \( p(t') = v_L \), it follows that \( p(t) < v_L \), which is a contradiction.

Now suppose \( q(t) < q^* \), but \( p(t) = v_L \). Together, they imply that the buyer at \( t \) offers \( c_H \), which is a strictly lower probability than \( \gamma_L(s_n^*) \sigma_B^* + 1 - \Gamma_L(s_n^*) \). In particular, the buyer’s probability of offering \( c_H \) cannot exceed \( 1 - \Gamma_L(s_n^*) \), because the payoff from doing so is negative conditional on signal \( s^* \). If \( \tilde{p}(t) \leq 0 \), then \( p(t) < v_L \), because \( r(p(t) - c_L) \leq \lambda(1 - \Gamma_L(s_n^*))(c_H - p(t)) + \tilde{p}(t) \), while \( v_L \) satisfies \( r(v_L - c_L) = \lambda(1 - \Gamma_L(s_n^*) + \gamma_L(s^*) \sigma_B^*)(c_H - v_L) \). This is a contradiction.
If $\hat{p}(t) > 0$, then, by the continuity of both $p(\cdot)$ and $q(\cdot)$, there exists $t'$ such that $q(t') < q^*$, but $p(t') > v_L$. We showed above that this cannot arise.

(2) If $q(t) > q^*$, then $p(t) > v_L$.

Suppose $q(t) > q^*$, but $p(t) < v_L$. We first show that there exists $t' \in (t, \infty)$ such that $p(t') = v_L$. Suppose not, that is, $p(t') < v_L$ for any $t' \geq t$. This implies that $q(\cdot)$ keeps increasing. Since $q(t) \in [0, 1]$ for any $t$, this means that there exists $q^\infty \in (q(t), 1]$ such that $q(\cdot)$ converges to $q^\infty$. Since the low type trades whenever a buyer arrives, the convergence can occur only when the high type trades with almost probability 1. This, in turn, implies that in the long run, each buyer offers $c_H$ with probability almost 1, regardless of his signal. But then the low-type seller’s reservation price becomes arbitrarily close to $\frac{rc_L + \lambda c_H}{r + \lambda}$. This is a contradiction, because $\frac{rc_L + \lambda c_H}{r + \lambda}$ is strictly larger than $v_L$ under Assumption 1. This establishes the claim that there must exist $t' > t$ with $p(t') = v_L$. Let $t'$ be the smallest value such that $p(t') = v_L$. Since $p(x) < v_L$ for any $x \in (t, t')$, $q(\cdot)$ cannot decrease on $(t, t')$. Therefore, $q(x) > q^*$ for any $x \in (t, t')$. Let $t'' \equiv t' - \epsilon$ for $\epsilon$ positive, but sufficiently small (so that $t'' > t$). Then, for any $x \in (t'', t')$, the buyer must offer $c_H$ with probability 1 whenever his signal is weakly above $s_{n^*}$. To see this, notice that, since $x$ is close to $t'$, $p(x)$ is close to $v_L$. Therefore, when the buyer’s signal is $s_{n^*}$, his expected payoff by offering $p(x)$ is also close to 0. To the contrary, his expected payoff by offering $c_H$ is bounded away from 0, because $q(x) \geq q(t) > q^*$ (recall that the payoff is equal to 0 if $q(x) = q^*$). This establishes the claim that buyers must offer $c_H$ with probability 1 when their signal weakly exceeds $s^*$. But then $p(x) > v_L$, because the buyers on $(x, t')$ offer $c_H$ at least with probability $1 - \Gamma_L(s_{n^*})$, while the low-type seller’s reservation price at $t'$ is equal to $v_L$ (recall that the low-type seller’s reservation price is equal to $v_L$ if every buyer offers $c_H$ with probability $\gamma_L(s_{n^*})\sigma_B + 1 - \Gamma_L(s_{n^*})$). This is a contradiction.

Now suppose $q(t) > q^*$, but $p(t) = v_L$. In this case, the low type does not necessarily accept $p(t)$ with probability 1. Therefore, $q(\cdot)$ is not necessarily increasing. However, we do know that the buyer would offer $c_H$ with probability 1 whenever his signal is weakly above $s_{n^*}$: since $p(t) = v_L$, the buyer with signal $s_{n^*}$ obtains zero expected payoff by offering $p(t)$, while his expected payoff by offering $c_H$ is strictly positive given that $q(t) > q^*$. If $\hat{p}(t) \geq 0$, then $p(t) > v_L$, because $r(p(t) - c_L) \geq \lambda(1 - \Gamma_L(s_{n^*}))(c_H - p(t)) + \hat{p}(t)$, while $v_L$ satisfies $r(v_L - c_L) = \lambda(1 - \Gamma_L(s_{n^*}) + \gamma_L(s^*)\sigma_B(c_H - v_L))$. If $\hat{p}(t) < 0$, then there exists $t' > t$ such that $q(t') > q^*$, but $p(t') < v_L$. We showed above that this cannot be the case.

(3) If $q(t) = q^*$, then $p(t) = v_L$.

Suppose $q(t) = q^*$ but $p(t) < v_L$. Since buyers’ beliefs would be increasing, there would exist $t'$ such that $q(t') > q^*$, while $p(t') < v_L$, which cannot be the case. Symmetric arguments lead to a contradiction for the case where $p(t) > v_L$.

The above result implies that, in any equilibrium, there is a one-to-one correspondence between time $t$ and buyers’ beliefs $q$. Therefore, the low-type seller’s reservation price, which is formally a function of his time-on-the-market, can be expressed as a function of buyers’ beliefs. For the same reason, buyers’ offer strategies can also be expressed as a function of $q$. Let $\bar{s}(q)$ denote the cutoff signal that a buyer with prior belief $q$ uses. We now establish that $\hat{p}(q)$ is increasing, while $\bar{s}(q)$ is non-increasing. We argue this separately for the two ranges of beliefs: $q \in (q^*, \bar{s}_1)$ and $q < q^*$.

First consider $q \in (q^*, \bar{s}_1)$. Then, by Lemma 2, $\min\{\hat{p}(q), v_L\} = v_L$. The weak monotonicity of $\bar{s}(q)$ then follows from equation (7). This, in turn, implies the strict monotonicity of $\hat{p}(q)$ via equation (8).
Next, consider \( q < q^* \). If \( \tilde{p}(q) \) is monotone in \( q \), then the monotonicity of the cutoff signals immediately follows. Yet, this monotonicity is not a priori clear, because if \( \tilde{p}(\cdot) \) is decreasing, then offering \( \tilde{p}(q) \) could be relatively more attractive when \( q \) is higher, and thus the buyer could be more reluctant to offer \( c_H \), which, in turn, could be consistent with decreasing \( \tilde{p}(\cdot) \). The next lemma rules out this possibility by establishing that \( p(t) \) is increasing over time whenever \( q(t) < q^* \).

**Lemma 3** In any equilibrium, if \( q(t) < q^* \), then \( p(\cdot) \) is strictly increasing in \( t \).

**Proof.** Suppose there exists \( t \) such that \( q(t) < q^* \), but \( \dot{p}(t) \leq 0 \). By Lemma 2, \( p(t) \) is strictly smaller than \( v_L \) and eventually converges to \( v_L \). Since \( p(\cdot) \) is also continuous, there exists \( t' \) such that \( t' > t \) and \( p(t') = p(t) \). Without loss of generality, assume that \( p(x) \leq p(t) \) for any \( x \in (t, t') \) and \( \dot{p}(x) > 0 \) for any \( x > t' \) such that \( q(x) < q^* \) (if \( p(\cdot) \) is not strictly increasing until it reaches \( v_L \), there always exist \( t \) and \( t' \) that satisfy these properties). For \( x \in (t, t') \), \( p(x) \leq p(t') \), while \( q(x) < q(t') \). This implies that the cutoff signal used by the buyer at \( x \in (t, t') \) must be at least as large as that used by the buyer at \( t' \). To the contrary, whenever \( x > t' \), \( p(x) > p(t') \) and \( q(x) > q(t') \). Therefore, the cutoff signal used by the buyer at \( x > t' \) must be no larger than that used by the buyer at \( t' \). Combining these observations with the fact that \( p(\cdot) \) is strictly increasing from \( t' \), it follows that \( p(t) < p(t') \), which is a contradiction. ■

Let us summarize how all the components we have established so far prove the equilibrium uniqueness.

First, Section 3.3.1 establishes that if buyers’ beliefs are to remain constant at some level less than \( \tilde{q}_t \), this level must be \( q^* \) as defined there. Lemma 2 implies that in any equilibrium buyers’ beliefs must be decreasing if \( q(t) > q^* \), while increasing if \( q(t) < q^* \). Since \( q(t) \) is continuous in \( t \), buyers’ beliefs cannot “jump over” \( q^* \) and must remain constant once they reach \( q^* \).

If \( \tilde{q} > q^* \), then the low-type seller, as well as the high-type seller, accepts only \( c_H \), until buyers’ beliefs reach \( q^* \). This uniquely pins down buyers’ behavior for \( q > q^* \) via equation (7). Then, the low-type seller’s reservation prices for this range of beliefs are determined by equation (3).

Finally, if \( \tilde{q} < q^* \), then the uniqueness argument requires an extra step, since in this case buyers’ equilibrium offer strategies cannot be pinned down independently of the low-type seller’s reservation price. In this case, a crucial step is Lemma 3 which establishes that \( p(t) \) must be increasing over time. Given this result, it is clear that any equilibrium must be constructed as in Section 3.3.2. The equilibrium uniqueness then follows from the fact that the construction yields a unique strategy profile. ■

**Proof of Proposition 1**

For each \( n = 1, \ldots, N - 1 \), let

\[
\lambda_n^* = \frac{r(v_L - c_L)}{(1 - \Gamma_L(s_n))(c_H - v_L)} = \frac{\rho_L}{1 - \Gamma_L(s_n)}.
\]

In addition, \( \lambda_0^* = \rho_L \). In words, \( \lambda_{n-1}^* \) is the smallest value of \( \lambda \) for which \( s_n \) serves as the cutoff signal on the stationary path.

First, observe that \( \tilde{q}_n \) is independent of \( \lambda \) when \( \lambda > \lambda_{n-1}^* \). This follows immediately, because
for such $\lambda$, $n^* \leq n$ and, therefore, $\overline{q}_n$ is determined by

$$
\overline{q}_n \frac{\gamma(s_n)}{\gamma_H(s_n)} \frac{c_H - v_L}{v_H - c_H}.
$$

Next, we show that $\overline{q}_n$ is strictly decreasing in $\lambda$ while $\lambda < \lambda_{n-1}^*$. We prove this result by a mathematical induction on $k$ where $\lambda \in (\lambda_{n-k-1}^*, \lambda_{n-k})$.

First consider $k = 1$, that is, $\lambda \in (\lambda_{n-2}^*, \lambda_{n-1}^*)$, which implies that $q^* = \overline{q}_{n-1}$. In equilibrium,

$$
\overline{q}_{n-1} = \overline{q}_n \frac{e^{-\lambda(1-\Gamma_H (s_{n-1}))T} \overline{q}_{n-1}}{1 - \overline{q}_n} \frac{e^{-\lambda T \overline{q}_{n-1}}}{1 - \overline{q}_n} = \overline{q}_n e^{\lambda(1-\Gamma_H (s_{n-1}))T} \overline{q}_{n-1}.
$$

Suppose $\overline{q}_n$ weakly increases in $\lambda$. Then, by equation (10) and the fact that $\overline{q}_{n-1}$ is independent of $\lambda$, $\lambda T (\overline{q}_n, \overline{q}_{n-1})$ must decrease. But then $\hat{p}(\overline{q}_n)$ strictly increases (see equation (8)), and thus equation (9) cannot be satisfied.

Next, take $k > 1$ and suppose that $\overline{q}_n$ strictly decreases in $\lambda$ over $(\lambda_{n-k-1}^*, \lambda_{n-k})$ and $\overline{q}_{n-1}$ strictly decreases in $\lambda$ over $(\lambda_{n-k-2}^*, \lambda_{n-k-1}^*)$. To complete the argument, we show that $\overline{q}_n$ also strictly decreases in $\lambda$ over $(\lambda_{n-k-2}^*, \lambda_{n-k-1}^*)$. Toward a contradiction, suppose that $\overline{q}_n$ weakly increases in $\lambda$ over $(\lambda_{n-k-2}^*, \lambda_{n-k-1}^*)$. Then, by equation (10) and the fact that $\overline{q}_{n-1}$ strictly decreases in $\lambda$, $\lambda T (\overline{q}_n, \overline{q}_{n-1})$ strictly decreases. This leads to the same contraction as in the previous case. Notice that the same argument applies even if a marginal increase of $\lambda$ changes the stationary cutoff belief (i.e., an increase of $\lambda$ around $\lambda_{n-k}^*$). This is because equation (9) always holds and, despite the discrete change of the stationary belief, the low-type seller’s reservation price continuously changes (see Section 3.4.2).

For the result on the low-type seller’s expected payoff, denote by $\psi_L(t)$ the rate at which the low-type seller receives offer $c_H$ at time $t$. In addition, we assume that $\hat{q} \in (\overline{q}_{n+1}, \overline{q}_n)$. There are three cases to consider.

(i) $\hat{q} \geq q^*$ (i.e., $n \leq n^*$) even before an increase in $\lambda$.

From the equilibrium structure,

$$
\psi_L(t) = \begin{cases} 
\lambda(1 - \Gamma_L(s_n)), & \text{if } t \leq T(\hat{q}, \overline{q}_{n+1}) \\
\lambda(1 - \Gamma_L(s_{n+1})), & \text{if } t \in (T(\hat{q}, \overline{q}_{n+1}), T(\hat{q}, \overline{q}_{n+2})), \\
\rho_L, & \text{if } t > T(\hat{q}, q^*).
\end{cases}
$$

$\psi_L(t)$ is a decreasing step function. As $\lambda$ increases, each $\lambda(1 - \Gamma_H(s_{n+k}))$ increases, while both $\lambda T(\hat{q}, \overline{q}_{n+1})$ and $\lambda T(\overline{q}_{n+k}, \overline{q}_{n+k+1})$ stay constant, as explained in the main text. This means that the low-type seller receives $c_H$ more frequently at earlier times as $\lambda$ increases. In other words, the distribution of the random time at which the low-type seller receives offer $c_H$ decreases in the sense of first-order stochastic dominance. Due to discounting, this clearly benefits the low-type seller. Notice that this argument is independent of whether $n^*$ changes or not. If $n^*$ drops by 1 due to an increase in $\lambda$, $\psi_L(t)$ takes one more step, but this does not affect the ranking argument above, because $\psi_L(t)$ always stays above $\rho_L$.

(ii) $\hat{q} < q^*$ before (with $\lambda$), but $\hat{q} \geq q^*$ after (with $\lambda'$).

The result is straightforward because $\hat{p}(\hat{q})$ is smaller than $v_L$ before the change, but becomes greater than $v_L$ after the change.
(iii) $\hat{q} < q^*$ even after an increase in $\lambda$.

In this case,

$$
\psi_L(t) = \begin{cases} 
\lambda(1 - \Gamma_L(s_n)), & \text{if } t \leq T(\hat{q}, \bar{q}_n) \\
\lambda(1 - \Gamma_L(s_{n-1})), & \text{if } t \in (T(\hat{q}, \bar{q}_n), T(\hat{q}, \bar{q}_{n-1})) \\
\rho_L, & \text{if } t > T(\hat{q}, q^*).
\end{cases}
$$

This is an increasing step function. As $\lambda$ increases, $\lambda T(\hat{q}, \bar{q}_{n-k})$ decreases (because $\bar{q}_{n-k}$ decreases), while each value $\lambda(1 - \Gamma_L(s_{n-k}))$ increases. As in the first case, this means that the distribution of the random time at which the low-type seller receives offer $c_H$ decreases in the sense of first-order stochastic dominance, which clearly increases the low-type seller’s expected payoff.

For the results on each seller type’s time to trade, we begin by establishing that the trading rate of each type on the stationary path increases in $\lambda$.

**Lemma 4** $\rho_H$ is strictly increasing in $\lambda$ if $\lambda < \lambda^*_n$ and independent of $\lambda$ if $\lambda > \lambda^*_n$.

**Proof.** Suppose $\lambda \in (\lambda^*_n, \lambda^*_n)$. In this case,

$$
\rho_H = \lambda(1 - \Gamma_H(s_n) + \gamma_H(s_n)\sigma^*_B),
$$

where $\sigma^*_B$ is the value that satisfies

$$
\lambda(1 - \Gamma_L(s_n) + \gamma_L(s_n)\sigma^*_B) = \rho_L = \frac{r(v_L - c_L)}{c_H - v_L}.
$$

From the latter equation,

$$
\frac{d\sigma^*_B}{d\lambda} = \frac{1 - \Gamma_L(s_n) + \gamma_L(s_n)\sigma^*_B}{\lambda\gamma_L(s_n)}.
$$

Therefore,

$$
\frac{d\rho_H}{d\lambda} = 1 - \Gamma_H(s_n) + \gamma_H(s_n)\sigma^*_B + \lambda\gamma_H(s_n)\frac{d\sigma^*_B}{d\lambda} = \gamma_H(s_n) \left( \frac{1 - \Gamma_H(s_n)}{\gamma_H(s_n)} - \frac{1 - \Gamma_L(s_n)}{\gamma_L(s_n)} \right) \geq 0,
$$

with strict inequality holding if and only if $\lambda < \lambda^*_n$. The last inequality is due to the monotone likelihood ratio property, which implies the hazard ratio dominance property.

The global monotonicity follows from the fact that $\rho_H$ is continuous even at each $\lambda^*_n$, which is, in turn, due to the fact that $\sigma^*_B$ converges to 0 as $\lambda$ increases to $\lambda^*_n$, while it approaches 1 as $\lambda$ decreases to $\lambda^*_n-1$.

Denote by $\phi_a(t)$ the rate at which the type-$a$ seller trades at time $t$. We first consider the high-type seller.

(i) $\hat{q} > q^*$
In this case,
\[ \phi_H(t) = \begin{cases} 
\lambda(1 - \Gamma_H(s_n)), & \text{if } t \leq T(\hat{q}, \hat{q}_{n+1}) \\
\lambda(1 - \Gamma_H(s_{n-1})), & \text{if } t \in (T(\hat{q}, \hat{q}_{n+1}), T(\hat{q}, \hat{q}_{n+2})) \\
\ldots & \\
\rho_H, & \text{if } t > T(\hat{q}, q^\ast).
\end{cases} \]

\(\phi_H(t)\) is a decreasing step function. As \(\lambda\) increases, each \(\lambda(1 - \Gamma_H(s_{n+k}))\) (and \(\rho_H\)) increases, while both \(\lambda T(\hat{q}, \hat{q}_{n+1})\) and \(\lambda T(\hat{q}_{n+k}, \hat{q}_{n+k+1})\) stay constant. This means that the high-type seller receives \(c_H\) more frequently at earlier times as \(\lambda\) increases: observe that the total probability that trade takes place while \(q(t)\) lies in the interval \((\hat{q}_{n+k}, \hat{q}_{n+k+1})\) remains constant, but the time it takes \(q(t)\) to travel through the interval shrinks. This clearly makes the high-type seller trade faster (in the sense of first-order stochastic dominance). Notice that this argument is independent of whether \(n^\ast\) changes or not. If \(n^\ast\) drops by 1 due to an increase in \(\lambda\), \(\phi_H(t)\) takes one more step, but this does not affect the ranking argument above, because \(\phi_H(t)\) always stays above \(\rho_H\).

(ii) \(\hat{q} < q^\ast\)

In this case,
\[ \phi_H(t) = \begin{cases} 
\lambda(1 - \Gamma_H(s_n)), & \text{if } t \leq T(\hat{q}, \hat{q}_n) \\
\lambda(1 - \Gamma_H(s_{n-1})), & \text{if } t \in (T(\hat{q}, \hat{q}_n), T(\hat{q}, \hat{q}_{n-1})) \\
\ldots & \\
\rho_H, & \text{if } t > T(\hat{q}, q^\ast).
\end{cases} \]

This is an increasing step function. As \(\lambda\) increases, \(\lambda T(\hat{q}, \hat{q}_{n-k})\) decreases (because \(\hat{q}_{n-k}\) decreases), while each value \(\lambda(1 - \Gamma_H(s_{n-k}))\) increases. As for the first case, this clearly speeds up trade of the high type.

(iii) An increase in \(\lambda\) that lowers \(q^\ast\) from above \(\hat{q}\) to below \(\hat{q}\)

The result is immediate from the following observations: before the increase, \(\phi_H(t) \leq \rho_H\) for any \(t\), while after the increase, \(\phi_H(t) \geq \rho_H\) for any \(t\). In addition, by Lemma \[\square\] the latter \(\rho_H\) is no smaller than the former.

We now consider the low-type seller and show that the same result holds if \(\hat{q} > q^\ast\) or \(\hat{q} < q^\ast\) but \(\lambda > \lambda_{N-1}^\ast\).

(i) \(\hat{q} > q^\ast\)

This part of the proof is identical to the corresponding one for the high-type seller.

(ii) \(\hat{q} < q^\ast\) and \(\lambda > \lambda_{N-1}^\ast\) (which implies that \(n^\ast = N\))

In this case, the low-type seller’s trading rate is given as follows:
\[ \phi_L(t) = \begin{cases} 
\lambda, & \text{if } t \leq T(\hat{q}, q^\ast) \\
\rho_H, & \text{if } t > T(\hat{q}, q^\ast).
\end{cases} \]

Since \(q^\ast\) is fixed, \(\lambda T(\hat{q}, q^\ast)\) is independent of \(\lambda\). Combining this with the fact that \(\phi_L(t)\) shifts leftward and upward as \(\lambda\) increases, it follows that the distribution of the low-type seller’s time to trade decreases in \(\lambda\) in the sense of first-order stochastic dominance.

**Proof of Lemma \[\square\].** By definition, there exists an \(N' \times N\) matrix \(M\) such that for each
\( a = H, L, \)

\[ \gamma'_a(s'_1) = \sum_{j=1}^{N} m_{1j} \gamma_a(s_j), \]

and

\[ \gamma'_a(s'_{N'}) = \sum_{j=1}^{N} m_{N'j} \gamma_a(s_j). \]

Since \( \gamma_H(s_n)/\gamma_L(s_n) \) is strictly increasing in \( n \),

\[ \frac{\gamma'_H(s'_1)}{\gamma'_L(s'_1)} = \frac{\sum_{j=1}^{N} m_{1j} \gamma_H(s_j)}{\sum_{j=1}^{N} m_{1j} \gamma_L(s_j)} \geq \frac{\gamma_H(s_1)}{\gamma_L(s_1)}, \]

while

\[ \frac{\gamma'_H(s'_{N'})}{\gamma'_L(s'_{N'})} = \frac{\sum_{j=1}^{N} m_{N'j} \gamma_H(s_j)}{\sum_{j=1}^{N} m_{N'j} \gamma_L(s_j)} \leq \frac{\gamma_H(s_N)}{\gamma_L(s_N)}. \]

The result follows by combining these inequalities with equation (7).

\[ \Box \]

**Proof of Proposition 2**

Fix \( \hat{q} < (c_H - v_L)/(v_H - v_L) \), and consider an inspection technology \( \Gamma \) such that \( \hat{q} < \bar{q}_N \). It is straightforward to construct such an inspection technology: the inequality holds whenever \( \Gamma \) is sufficiently uninformative. See also the online appendix (Appendix B). Suppose \( \Gamma' \) is less informative than \( \Gamma \). Lemma 1 implies that \( \bar{q}_{N'} \) is greater under \( \Gamma' \) than under \( \Gamma \), and thus the inequality \( \hat{q} < \bar{q}_N \) is preserved under \( \Gamma' \).

When \( \hat{q} < \bar{q}_N = q^* \), the low-type seller’s expected payoff at time 0 is given by

\[ p(\hat{q}) = \left(1 - e^{-rT(\hat{q}, \bar{q}_N)}\right) C_L + e^{-rT(\hat{q}, \bar{q}_N)} v_L, \]

where

\[ \bar{q}_N = \frac{\hat{q}}{\hat{q} + (1 - \hat{q})e^{-\lambda T(\hat{q}, \bar{q}_N)}}. \]

The result for the low-type seller’s expected payoff follows from the fact that \( \bar{q}_N \) is smaller under \( \Gamma \) than under \( \Gamma' \) (Lemma 1), and thus \( T(\hat{q}, \bar{q}_N) \) is also smaller under \( \Gamma \) than under \( \Gamma' \).

For the times to trade, first notice that the trading rate of each seller type on the stationary path is equal to

\[ \rho_H = \lambda \gamma_H(s_N) \sigma_B^* = \frac{r(v_L - c_L) \gamma_H(s_N)}{c_H - v_L} \gamma_L(s_N) = \rho_L \frac{\gamma_H(s_N)}{\gamma_L(s_N)}. \]

Since the maximal likelihood ratio \( \gamma_H(s_N)/\gamma_L(s_N) \) is greater under \( \Gamma \) than under \( \Gamma' \), it follows that \( \rho_H \) is also larger under \( \Gamma \) than under \( \Gamma' \).

The high-type seller never trades until \( T(\hat{q}, \bar{q}_N) \) and trades at rate \( \rho_H \) thereafter. Since \( T(\hat{q}, \bar{q}_N) \) is smaller, while \( \rho_H \) is higher, under \( \Gamma \) than under \( \Gamma' \), it is clear that the time to trade \( \tau_H \) is smaller under \( \Gamma \) than under \( \Gamma' \) in the sense of first-order stochastic dominance.

The low-type seller trades at rate \( \lambda \) until \( T(\hat{q}, \bar{q}_N) \) and trades at rate \( \rho_H \) thereafter. Although
this change cannot be ranked in terms of first-order stochastic dominance,

\[ E[\tau_L] = \int_0^{T(\hat{q}, \bar{q}_N)} td(1 - e^{-\lambda t}) + e^{-\lambda T(\hat{q}, \bar{q}_N)} \left( T(\hat{q}, \bar{q}_N) + \frac{1}{\rho_H} \right) = \frac{1 - e^{-\lambda T(\hat{q}, \bar{q}_N)}}{\lambda} + \frac{e^{-\lambda T(\hat{q}, \bar{q}_N)}}{\rho_H}. \]

Notice that \( \rho_H = \rho_L \gamma_H(s_N)/\gamma_L(s_N) \), while

\[ e^{-\lambda T(\hat{q}, \bar{q}_N)} = \frac{\hat{q}}{1 - \hat{q}} \frac{1 - \bar{q}_N}{\bar{q}_N} = \frac{\hat{q}}{1 - \hat{q}} \frac{\gamma_H(s_N)}{\gamma_L(s_N)} \frac{c_H - v_L}{v_H - c_H}. \]

Therefore, the expression shrinks to

\[ E[\tau_L] = \frac{1 - e^{-\lambda T(\hat{q}, \bar{q}_N)}}{\lambda} + \frac{\hat{q}}{1 - \hat{q}} \frac{c_H - v_L}{v_H - c_H} \frac{1}{\rho_L}. \]

Since \( T(\hat{q}, \bar{q}_N) \) is smaller under \( \Gamma \) than under \( \Gamma' \), while \( \rho_L \) is independent of the inspection technology, \( E[\tau_L] \) is clearly smaller under \( \Gamma \) than under \( \Gamma' \).

The second set of results is clear from Lemma 1 and the equilibrium structure. 

References


