Learning in the stock market and credit frictions

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Abstract

Standard models of credit frictions do not typically generate substantial amplification and propagation of macroeconomic shocks. I argue that this is directly related to the failure of these models to generate sufficient endogenous volatility in asset prices. I introduce learning about the stock market to a model in which firms borrow against the market value of their collateralised assets. A strong positive feedback loop emerges between beliefs, asset prices and profits which reinforces the learning dynamics. Small shocks on either the supply or demand side of the economy then translate into large and persistent aggregate fluctuations.

1 Introduction

The global financial and economic turmoil of recent years has led to at least two insights: credit frictions seem to matter much more than previously thought; and the ability of financial markets to optimally forecast movements in asset prices has to be questioned. These two aspects are of course related, as large unexpected declines in asset prices can force a decline in economic activity insofar as agents rely on these assets as collateral to finance their expenditures.

It is therefore natural to examine asset price volatility in theoretical models of financial frictions. These models have become popular to work with at least since the work of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). However, they typically build on standard business cycle models which are notorious for their failure to capture key asset price properties, including price volatility. I argue that this failure is directly related to the finding (e.g. Cordoba and Ripoll 2004) that standard credit friction models do not generate as much amplification and propagation of macroeconomic shocks as had been hoped in the early stages of research in this area. If the behaviour of asset prices is

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improved, then this will also increase the importance of credit frictions as an amplification mechanism, in contrast to them being a mere source of exogenous “deleveraging shocks” (Jermann and Quadrini 2012).

In this paper, I develop a model where firms borrow against the market value of their assets. I do not use the book value or the liquidation value of these assets but the going-concern value, which is generally higher than the former in the presence of financing constraints. The going concern value includes the price of equity, and so a high equity valuation means easier access to credit. This idea is already implicit in most costly state verification models (e.g. Cooley and Quadrini 2001), where the stock market value equals the value function of the firm, which in turn determines the likelihood of default. In a limited commitment framework such as the one I am using, the dependency on the going-concern value can be rationalised through a default procedure in which defaulting firms can be either liquidated or restructured and sold off to other investors. The expected equity resale value after restructuring then determines how much funds lenders are willing to extend to the firm in operation.

Equity prices in turn have been extensively covered in the finance literature, and the failure of standard macroeconomic models to capture excess volatility and other “puzzles” is well documented. Many ways have been proposed to resolve these puzzles. The approach I am taking follows Adam and Marcet (2011) who have shown that simple models of adaptive learning do quite well in matching aggregate stock price behaviour. Introducing their learning mechanism creates a powerful feedback loop between prices, beliefs and fundamentals which enhance the learning dynamics of their partial equilibrium setup. At the same time, it serves as a powerful amplification mechanism for any kind of shocks that increases firms’ profits. An increase in firms’ profits causes optimism in the stock market, thereby relaxing credit constraints. This means firms can increase their investment and become even more profitable. This further raises stock market prices, and so forth. A calibrated version of the model is shown to amplify and propagate both demand and supply shocks.

Section 2 discusses the related literature. Section 3 presents some empirical evidence on the relationship between stock market valuation, investment and credit constraints, as well as the plausibility of replacing rational expectations with learning in stock market pricing. Section 4 presents the model. Section 5 describes the calibration strategy and discusses simulation results. Section 6 presents several extensions of the model. Section 7 concludes.

2 Related literature

On the learning side, the present paper is primarily inspired by Adam and Marcet (2011). They show that learning about stock market returns in an otherwise standard Lucas asset pricing framework can predict a surprising number of asset price “puzzles”, including
return predictability and excess volatility. Johannes, Lochstoer, and Mou (2010) conduct a more involved, but similar exercise in a finance context, while Benhabib and Dave (2011) show theoretically that learning can lead to fat-tailed return distributions even if the underlying shocks are thin-tailed. But most papers take the dividend stream paid by the assets as exogenous, and so they lack the interaction between prices and fundamentals present in the model of this paper. Contributions towards asset price learning in a production economy have been made, among others, by Caputo, Medina, and Soto (2010), Milani (2011), Adam, Kuang, and Marcet (2011) and Kuang (2013). Milani uses learning about asset prices in a standard New-Keynesian model, and Caputo et al. perform a similar exercise in a model with financial frictions. These authors take a very comprehensive view of the learning process, meaning that many forward-looking parameters have to be jointly estimated in the perceived law of motion of every forward-looking variable in the model. This makes it very difficult to assess the effect of asset price learning from learning about other quantities, such as productivity. In my model, only the growth rate of stock prices needs to be learned, while agents have rational expectations on all other variables. This adds only a single state variable to the model, and also permits to solve it with standard log-linearisation techniques. Adam et al. and Kuang employ learning to generate excess real volatility in a model of housing and the Kiyotaki-Moore model, respectively. However, in their analysis, adding a production side dampens rather than amplifies the dynamics of the learning mechanism.

This paper also connects to the literature on sunspots, insofar as it affirms a strong role for changes in expectations for the business cycle. The ingredients that lead to sunspots are often similar to the ones needed for amplification in models of adaptive learning. As a recent example, Farmer (2012b) builds a model of labour market search where expectations about the stock market drive market tightness; while Farhi and Tirole (2012) and Miao and Wang (2011) explore the possibility of multiple asset price equilibria in the presence of liquidity constraints. The outstanding theoretical advantage of sunspots over learning is that they withstand the Lucas critique. However, the indeterminacy in these models is inherently unresolvable even in the long run - “optimism” and “pessimism” can possibly persist indefinitely. In my model, even though expectations change, the economy always returns to the steady state in the long run.

Two recent contributions by Miao, Wang, and Xu (2012) and Liu, Wang, and Zha (2013) share with this paper the emphasis of asset price fluctuations as a driver of access to credit. Miao et al. look at stock prices in a setup similar to the one in the present paper, while Liu et al. look at commercial real estate prices. However, in order to generate the asset price fluctuations, they need to feed their models a series of large exogenous “price shocks”. While their analysis reveals the importance of asset prices in credit frictions, it does not solve the problem of lacking amplification and propagation of shocks.

Finally, the paper also touches on the literature on news shocks, which examines which modifications to the neoclassical growth model can generate positive comovements in con-
sumption, investment and employment in response to changes in expectations. Jaimovich and Rebelo (2009) identify the behaviour of the wealth effect on labour supply as a key determinant. I employ their preference specification to rule out countercyclical employment responses in my model.

3 Some empirical evidence

3.1 Effect of the stock market on investment and access to credit

There is substantial evidence that movements in the stock market predict investment, as documented by Barro (1990). Blanchard, Rhee, and Summers (1993) pointed out that this predictive power does not necessarily establish causation though, because movements in stock prices might only reflect changes fundamentals that are the ultimate drivers of investment. However, Baker, Stein, and Wurgler (2003), using firm-level data, found evidence that stock market movements that cannot be attributed to fundamentals also raise investment, especially for younger firms. They conclude that this points to the presence of financial frictions which are relaxed when equity is cheap.

At the same time, there is positive comovement between aggregate stock market valuation and corporate spreads. Figure 1 plots the monthly P/D ratio of the S&P1500 against Moody’s baa-aaa corporate spread for the period between 1923 and 2012.\textsuperscript{1} The series display a clear negative correlation. There are several possible ways to explain this observation. First, both spreads and equity prices can be driven by changes in firms’ fundamental production conditions. For example, a positive productivity shock increases firms’ profits and make their equities more valuable, while at the same time reducing the probability of default. Second, a reduction of credit spreads can cause high equity valu-

\textsuperscript{1}The data are from Rob Shiller’s website and FRED.
ations. If firms have easier access to debt financing, they are less constrained, become more profitable and can pay out more dividends to shareholders. Third, spreads can be influenced by fluctuations in the valuation of equity, even if those fluctuations are not due to changes in firms’ fundamentals or credit market conditions. If a firm’s equity has high value, then the resale value of its assets in the event of default is high, too. This might make lenders more willing to extend loans to the firm.

I do not see these three directions of causality as a horserace. In fact, in the model of the next section, all of them will be present simultaneously. Rather, I want to convince the reader that the third channel exists; that an increase in stock market value which is not backed by fundamentals can by itself lead to easier access to credit and higher investment.

To this end, I construct a VAR in four variables using quarterly US data. The time series used are $s_t$, the baa-aaa Moody’s corporate spread; $P_t$, the S&P1500, of which S&P claims that it is representative for 9x% of US corporates; $z_t$, US total factor productivity adjusted for capacity utilisation as documented by Fernald (2009); and $I_t$, gross private non-residential fixed investment. I use capacity utilisation-adjusted TFP to disentangle “true” productivity changes from measured changes that are merely due to fluctuations in capacity utilisation, as discussed in Basu, Fernald, and Kimball (2006).

The data span the entire post-war period 1948Q2-2012Q4. All series are in levels. Although all the series except the spread are likely to contain a unit root, the specification in levels is still consistent if the lag length is higher than the level of integration of the system. The lag length of 4 is chosen by the Schwarz Bayesian Information Criterion. So the specification is

$$\begin{pmatrix}
  s_t \\
  P_t \\
  I_t \\
  z_t
\end{pmatrix} = X_t = \sum_{s=1}^{4} \psi^s X_{t-s} + c + \varepsilon_t$$

I construct impulse responses using a standard Cholesky decomposition with the ordering of the variables as presented in the equation above. Productivity lags investment, and both lag the credit and equity markets. Ordering the financial variables $s_t$ and $P_t$ is less straightforward, as both series exhibit strong comovement at shorter frequencies. Here, I will discuss the impulse responses from ordering $s_t$ before $P_t$. The resulting impulse responses for the reverse ordering are qualitatively similar.

Figure 2 presents impulse responses that aim at capturing the impact of productivity, credit spread, and stock market shocks.3 Starting with the top line, the VAR predicts that an unexpected increase in TFP has a positive impact on investment, but no significant impact on equity prices or credit spreads. On the second line, the impulse is a shock to the credit spread, which might be thought of as an increase in risk aversion or an

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2Data are from FRED and the San Francisco Fed.

3Confidence bands are constructed from bootstrapped residuals to take into account possible unit roots in the series.
uncertainty shock. In line with the findings of the uncertainty literature (e.g. Bloom 2009), an unexpected increase in the credit spread reduces investment and also, although insignificantly, aggregate productivity. Somewhat surprisingly, it has some positive impact on the price of equity, but the effect is insignificant.

Finally, an unexpected shock to stock market prices in the bottom line increases investment and markedly reduces the credit spread. It also predicts an increase in TFP. This could reflect news about future productivity (as analysed in the “news shock” literature, cf. Jaimovich and Rebelo 2009). However, the increase is only significant some 15 quarters ahead, which seems hard to reconcile with a large investment increase and a reduction in credit spreads immediately after the stock price shock. This leaves the possibility that the shock is capturing something other than a news shock, and that the stock market value has a direct impact on investment and access to external finance.

Of course, the evidence from this analysis is not conclusive. In general, VAR impulse responses are quite sensitive to omitted variables, and it is possible that the stock market shock in the current specification captures other omitted factors, such as shocks to aggregate demand.
3.2 Systematic bias in stock market expectations


The starting point is the well-known observation by Fama and French (1988) that medium-term stock market returns are predictable by the dividend yield. Figure 3 documents this fact again using the S&P1500 with monthly data from 1923-2012. It plots the current P/D ratio against the total return on the index four years ahead. The relationship is clearly negative. Such return predictability can be observed regardless of the return horizon and increases with its length.

By itself, this finding is not inconsistent with rational expectations. Indeed, the standard explanation goes that either risk aversion or actual risk is time-varying and predictably countercyclical. However, if such was the case, then investors’ expectations about future returns should be consistent with it. Thus, when the P/D ratio is high, investors should expect low future returns and vice-versa. However, when looking at survey data, as presented in Figure 4, this does not seem to be the case. The left panel of the figure plots the P/D ratio of the S&P1500 over time, as well as investors’ confidence in the future development of the stock market as measured by the qualitative index compiled at Yale University (“do you think the stock market is going to go up or down”). The correlation between the series should be highly negative, but is actually slightly positive - investors tend to be more optimistic about future prospects when the market is highly valued. Further, the right panel plots the detrended S&P1500 against the forecast errors of professional forecasters in the US Livingstone survey, which asks them to provide a
point estimate about the stock market 12 months from now. Under rational expectations, these forecast errors should be unpredictable, but in fact even professional forecasters tend to overestimate the future stock price level when the market is currently high and vice versa.

4 The model

This section constructs a fairly standard model of an economy with credit market frictions, where the market value of equity is crucial for firms’ access to external finance. I then introduce learning about this value and show how a feedback loop between fundamentals and expectations leads to considerable amplification of technology shocks, while the properties of the rational expectations version are not too different from a model in which credit frictions are absent altogether.

4.1 Building blocks

The economy is populated by two types of entrepreneurs, households, intermediate goods producers and retailers. Entrepreneurs own the intermediate producers which produce using capital and labour, but are borrowing-constrained. They are at the heart of the model. Households lend to intermediate producers, supply labour and consume differentiated final goods. They also own the retailers whose only role consists in transforming homogenous intermediates into differentiated final consumption goods.
Households

The representative household consumes, lends and supplies labour. They are endowed with preferences as in Jaimovich and Rebelo (2009), solving the following programme:

$$\max_{C_t, X_t, L_t, B_t} E \sum_{t=0}^{\infty} \beta^t \left( C_t - X_t L_t^{1+\phi} / (1 + \phi) \right)^{1-\theta} / (1 - \theta)$$

s.t. $P_t C_t = W_t L_t + B_t - (1 + i_{t-1}) B_{t-1} + \Pi_t^r$

$$\log X_t = \rho_x \log X_{t-1} + (1 - \rho_x) \log C_t$$

where $W_t$ is the nominal wage, $L_t$ is the amount of labour supplied, $B_t$ are funds loaned to intermediate firms, $i_{t-1}$ is the nominal interest rate on these loans, and $\Pi_t^r$ are profits from owning retail firms. Consumption $C_t$ itself represents a composite utility flow from of a variety of differentiated goods that takes a standard CES form:

$$C_t = \left( \int_0^1 \left( C_i^t \right)^{\frac{\sigma}{\sigma - 1}} \, di \right)^{\frac{\sigma - 1}{\sigma}}$$

These differentiated goods each sell at the relative price $p_i^t$. The price index $P_t$ of composite consumption consistent with utility maximisation and the demand function for good $i$ are then

$$P_t = \left( \int_0^1 \left( p_i^t \right)^{1-\sigma} \, di \right)^{\frac{\sigma}{1-\sigma}} ; C_i^t = \left( \frac{p_i^t}{P_t} \right)^{-\sigma} C_t$$

Finally, define $w_t = W_t / P_t$ as the real wage and $R_t = (1 + i_{t-1}) P_{t-1} / P_t$ as the ex-post real interest rate. Then the first order conditions of the household are:

$$w_t = X_t L_t^\phi$$

$$1 = \beta E_t \left( \frac{C_{t+1}^h - X_{t+1} L_{t+1}^\phi / (1 + \phi)}{C_t^h - X_t L_t^{1+\phi} / (1 + \phi)} \right)^{-\theta} R_{t+1}$$

Retailers

Retailers transform homogenous goods into differentiated final consumption goods using a one-for-one technology. The final good trades in a competitive market at the real price $q_t$ (expressed in units of the composite final consumption good). Each retailer enjoys market power in her output market though, and sets a nominal price $p_i^t$ for her production. A standard price adjustment friction à la Calvo means that a retailer cannot adjust her price with probability $\alpha_p$, which is independent across retailers and across time. Hence,
the retailer solves the following optimisation:

$$\max_{p_t} \sum_{s=0}^{\infty} \alpha_t^s Q_{t,t+s} \left( p_t^s - q_{t+s} P_{t+s} \right) C_{t+s}^i$$

s.t. (1)

where $Q_{t,t+s}$ is the nominal discount factor of households between time $t$ and $t + s$. Since all retailers that can reoptimise at $t$ are identical, they all choose the same price, which is denoted by $\tilde{p}_t$.

**Monetary policy**

I assume that monetary policy is given by a simple interest rate rule of the form

$$i_t = \phi \pi_t + \nu_t^m$$

where $1 + \pi_t = P_t / P_{t-1}$ is the inflation rate and $\nu_t^m$ is a policy disturbance.

**Intermediate firms**

Production of intermediate goods is realised by a continuum of firms $j \in [0,1]$. Firm $j$ enters period $t$ with capital $K_{t-1}^j$ and debt $B_{t-1}^j$. Capital is combined with labour to produce output

$$Y_t^j = \left( K_{t-1}^j \right)^{\alpha} \left( A_t L_t^j \right)^{1-\alpha}$$

where $A_t$ is aggregate productivity that follows a stationary AR(1) process. Output is sold competitively to retailers at the real price $q_t$ and labour is hired at the competitive real wage $w_t$. Hence, labour demand by firm $j$ follows

$$\max_{L_t^j} q_t Y_t^j - w_t L_t^j$$

Because of constant returns to scale, every firm in the economy chooses the same capital-labour ratio. This permits to write down an internal rate of return on capital. This rate of return is common to all firms:

$$R_t^k = \frac{q_t Y_t^j - w_t L_t^j + (1-\delta) K_{t-1}^j}{K_{t-1}^j} = \alpha q_t \left( \left( \frac{1-\alpha}{\alpha} \frac{q_t A_t}{w_t} \right)^{1-\alpha} + 1 - \delta \right)$$

Here, $\delta$ is the rate of capital depreciation. After production and capital depreciation, the firm has to repay its debt from the previous period, carrying real interest $R_t$. What is left is net worth $N_t^j$:

$$N_t^j = R_t^k K_{t-1}^j - R_t B_{t-1}^j$$

10
At this point, I assume that firms exit with an exogenous probability of $\gamma$. This prevents them from becoming financially unconstrained, as in Bernanke, Gertler, and Gilchrist (1999). When a firm dies, it pays out its entire net worth as dividends to its owner: $D_j^t = N_j^t$. I assume that raising equity is so costly that continuing firms cannot raise any equity. At the same time, new firms enter the economy at the rate $\gamma$. Of course, these new firms need to start with some initial equity to become operational. I therefore assume that entrepreneurs can inject equity into new firms, but can only do so up to a fraction $\omega$ of the existing equity in the economy. While this is certainly a crude assumption, it does capture the fact that equity issuance is procyclical (since aggregate net worth is procyclical), as documented in Covas and Den Haan (2011).

Both new and continuing firms then jointly choose capital, dividends and debt holdings subject to the flow-of-funds constraint

$$K_j^t + D_j^t = N_j^t + B_j^t$$

where $N_j^t = \begin{cases} N_j^t & \text{for continuing firms} \\ \omega \int_0^1 N_j^t dj & \text{for new firms} \end{cases}$

While firms are allowed to pay dividends, they will not choose to do so as long as they are financially constrained. As this will always be the case in the equilibrium I'm considering, $D_j^t = 0$ (see the Appendix for details). Moreover, in choosing their debt holdings, firms are subject to a financing constraint as follows.

The conventional way to introduce a borrowing friction is to assume that the lender can liquidate the firm in the event of default, so that the borrowing capacity of the firm is limited by the liquidation value of its assets. However, liquidation might not be the optimal strategy for the lender. Because assets have a higher value inside the firm, it can be more profitable to restructure some of the debt and sell the firm to a potential investor as a going concern, rather than to sell the assets outside of the firm. In this case, the borrowing limit of the firm is related to the market value of its equity.

More precisely, at the end of the period, but before the realisation of next period’s shocks, firm $j$ has the option to default on its debt. In this case, the value of the debt is renegotiated. If negotiations break down, the lender seizes the firm. Assume that the lender (in this model a household) does not have the abilities to run the firm. She can only liquidate or restructure it. If she liquidates the firms, a fraction $1 - \xi$ of the firm’s assets is lost, and obviously all debt is cancelled. The payoff from liquidation is therefore $\xi K_t$. If the lender restructures the firm instead, then the firm suffers the same loss of capital, but now the firm can be resold to another entrepreneur at the beginning of the next period. As in reality, restructuring also entails a partial loss of debt repayments. To preserve the aggregation property of the model, I impose that the debt cancelled is also a fraction $1 - \xi$ of initial debt. Thus, the restructured firm will have net worth $R_t^{k+1} \xi K_t^j - R_t^{k+1} \xi B_t^j = \xi N_t^{j+1}$. Because of the aggregation property, an entrepreneur is willing to buy this firm for exactly
the fraction $\xi$ of the equity value in the event of no default, $V_t$.

It can be shown that in the unique steady state of the model, restructuring is always preferred to liquidation by the lender. Intuitively, this is because capital is more valuable inside the firm than if it is sold on the market because of financial frictions. Another entrepreneur is always willing to buy a restructured firm from the lender at a price higher than the liquidation value.

The lender surplus from renegotiating the debt to $B_t^{*j}$ compared to seizing the firm is therefore $B_t^{*j} - \xi(B_t^j + V_t^j)$. I assume that the firm has all the bargaining power in the renegotiation process, so $B_t^{*j} = \xi(B_t^j + V_t^j)$. If the renegotiated debt fell short of the original value, then the lender would earn a return of less than $R_t$, and such a loan will not be made to the firm in the first place. Therefore, the borrowing limit is given by

$$B_t^j \leq \xi \left(B_t^j + V_t^j\right)$$

which can be rewritten as

$$B_t^j \leq \frac{\xi}{1 - \xi} V_t^j \tag{7}$$

Finally, the firm’s objective is to maximise expected dividend payments to its owners, the entrepreneurs. However, I focus on the case in which the borrowing constraint is always binding. Then, equations (6) and (7) already fully describe firm behaviour.

**Entrepreneurs and stock market pricing**

Entrepreneurs are distinct from households and their only role lies in the capacity to own intermediate firms. The representative entrepreneur is risk-neutral and discounts the future at a rate $\beta$ for which entrepreneurs and households discount future income at the same rate on the balanced growth path. She has to manage a portfolio of shares in firms. As described above, when a firm exits it pays out its net worth as dividend, and is replaced by a new firm which has the opportunity to raise a limited amount of equity. Let the set of exiting firms in period $t$ be $\Gamma_t$. Then the entrepreneur’s problem is

$$\max_{C_t, S_t} E \sum_{t=0}^{\infty} \beta^t C_t^e$$

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4Note that in principle, the lender could also fall short of a safe return of $R$ if the firm found itself with negative net worth $N_{t+1} < 0$ in some states, in which case the firm would need to default as well. But because $R_{t+1} \geq 1 - \delta$, this could only happen if $B_t^j / K_t^j > (1 - \delta) / R$, i.e. if leverage ratios are extremely high. This is not a problem in any reasonable calibration.

5This is not the same as having $\beta = \beta$ if the intertemporal elasticity of substitution is different for households and entrepreneurs and the balanced growth path has non-zero growth.
From this, the stock market pricing equation emerges as

\[ V_{jt} = \beta E_t \left[ N_{jt} I (j \in \Gamma_{t+1}) + V_{jt+1} I (j \notin \Gamma_{t+1}) \right] \]

**Asset pricing under rational expectations**

Because of constant returns to scale, firm value in equilibrium is proportional to current net worth \( \tilde{N}_{jt} \) (see the Appendix for a proof). In particular:

\[ V_{jt}^j = \frac{V_t}{\tilde{N}_t} N_{jt} \tag{8} \]

where \( V_t = \sum_{j} V_{jt}^j dj \) is the aggregate value of the stock market and \( \tilde{N}_t = \sum_{j} \tilde{N}_{jt} dj = (1 - \gamma + \gamma \omega) N_t \). The stock market itself has as its Euler equation

\[
V_t = \beta E_t \left[ \int_{j \in \Gamma_{t+1}} N_{jt}^j + \int_{j \notin \Gamma_{t+1}} V_{jt+1}^j \right]
\]

\[ = \beta E_t \left[ \gamma N_{t+1} + \frac{V_{t+1}}{\tilde{N}_{t+1}} \int_{j \notin \Gamma_{t+1}} N_{jt+1}^j \right] \tag{10} \]

\[ = \beta E_t \left[ \gamma N_{t+1} + \frac{1 - \gamma + \gamma \omega}{1 - \gamma + \gamma \omega} V_{t+1} \right] \tag{11} \]

This looks very much like a standard asset pricing equation, with a correcting term accounting for the fact that next year’s stock market value \( V_{t+1} \) includes some firms that are yet unborn at time \( t \).

**Asset prices under learning**

There are two financial assets in the economy, equities and bonds.

I introduce learning about the price of equities following Adam and Marcet (2011). From an empirical perspective, learning improves the behaviour of asset prices, leading to excess volatility and return predictability without the need to impose implausibly high rates of risk aversion or complex preference structures. Theoretically, it can be shown to be sustainable as a “minimal” departure from rational expectations in which agents behave rationally themselves, but are unsure about the behaviour of other agents.

\[ \text{In Miao and Wang (2011), which employ a model structure similar to mine, there can be multiple equilibria due to the presence of rational bubbles. The presence of these bubbles makes firm value non-proportional in net worth. The Appendix also establishes conditions under which such bubbles could also arise in the present model. For the calibrations I use, these are never satisfied.} \]
I assume that agents are able to form rational expectations about all variables in the economy\(^7\) except for prices in the stock and bond markets. In this sense, the departure from rational expectations is small. Agents’ perceived law of motion for the stock market is a random walk with a time-varying drift which can be thought of as the long-run growth rate of the stock market value:

\[
\begin{align*}
\log V_t &= \log V_{t-1} + \mu_t + \eta_t \\
\mu_t &= \mu_{t-1} + \nu_t \\
\begin{pmatrix} \varepsilon_t \\ \nu_t \end{pmatrix} &\sim \text{i.i.d.} \mathcal{N}\left( -\frac{1}{2} \begin{pmatrix} \sigma_\eta^2 & 0 \\ 0 & \sigma_\nu^2 \end{pmatrix} , \begin{pmatrix} \sigma_\eta^2 & 0 \\ 0 & \sigma_\nu^2 \end{pmatrix} \right)
\end{align*}
\]

(12)

(13)

(14)

In this situation, it is optimal for agents to use the Kalman filter to update their beliefs about the hidden state variable \(\mu_t\). Denote the beliefs of agents using information up to period \(t\) by \(\mathcal{P}_t\). I adopt the timing convention that in period \(t\), agents first carry out transactions using their prior belief \(\mathcal{P}_{t-1}\) and then calculate their posterior \(\mathcal{P}_t\) at the end of the period. The idea here is that prices and beliefs cannot be determined simultaneously because beliefs adjust only slowly to new information. Therefore, agents’ forecast of future stock market prices during period \(t\) is

\[
E_{t}^{\mathcal{P}_{t-1}} V_{t+1} = \exp (\hat{\mu}_{t-1}) V_t
\]

where \(\hat{\mu}_{t-1} = E_{t}^{\mathcal{P}_{t-1}} \mu_{t-1}\). The actual stock market price \(V_t\) is then determined by the entrepreneurs’ Euler equation (9), where the expectations operator \(E_t V_{t+1}\) is replaced by the belief \(E_{t}^{\mathcal{P}_{t-1}} V_{t+1}\).\(^8\):

\[
\begin{align*}
V_t &= \beta E_{t}^{\mathcal{P}_{t-1}} [\gamma N_{t+1}] + \beta \frac{1 - \gamma}{1 - \gamma + \gamma \omega} \exp (\hat{\mu}_{t-1}) V_t \\
&= \frac{\beta E_{t}^{\mathcal{P}_{t-1}} [\gamma N_{t+1}]}{1 - \beta \frac{1 - \gamma}{1 - \gamma + \gamma \omega} \exp (\hat{\mu}_{t-1})}
\end{align*}
\]

(15)

After observing this price, agents update their beliefs about permanent asset price growth. It can be shown (see Adam and Marcet 2010) that with an appropriate initial prior, beliefs evolve according to

\[
\hat{\mu}_t = (1 - g) \hat{\mu}_{t-1} + g \left( \log V_t - \log V_{t-1} + \frac{\sigma_\eta^2 + \sigma_\nu^2}{2} \right)
\]

(16)

where \(g\) is a function of the variances in (14).\(^9\) This law of motion for \(\hat{\mu}_t\) is fully optimal

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\(^7\)They also retain the belief about the proportionality of firm value to net worth (8).

\(^8\)Even though every agent in the economy is unaware that it is the Euler equation which determines \(\mathcal{P}_t\). See Adam and Marcet (2011) for a discussion of common knowledge assumptions under learning.

\(^9\)The appropriate prior is \(\mu_0 \sim \mathcal{N} (\hat{\mu}_0, \sigma_\mu^2)\) where \(g = \frac{1}{2} \left( -\frac{\sigma_\mu^2}{\sigma_\eta^2} + \sqrt{\frac{\sigma_\mu^2}{\sigma_\eta^2} + 4 \frac{\sigma_\nu^2}{\sigma_\eta^2} } \right)\)
in the Bayesian sense when agents hold the belief system (12)-(14).

Agents have to form expectations not only about the stock market, but also about interest rates and inflation in this model. Unlike most of the learning literature, I do not assume that all these expectations are also boundedly rational and subject to similar learning schemes. Instead, I employ the anticipated utility approach (see Kreps 1998; Cogley and Sargent 2006) to keep the departure from rational expectations minimal. Agents fully understand the entire model up to the behaviour of the stock market and make optimal forecasts conditional on their beliefs. For example, households are able to calculate their future consumption paths and corresponding lending rates for any path of shocks from the model equations, with the exception of the market-clearing condition (9), of which they are ignorant. Instead, they take the belief system (12)-(14) as the basis for predicting future stock prices, borrowing, investment, output and so forth.

**Equilibrium**

I focus on equilibria in which the borrowing constraint is always binding. In this situation, given a process for productivity $A_t$, monetary disturbances $\varepsilon_t^m$, and initial conditions $K_{-1}, B_{-1}, i_{-1}, P_{-1}, X_{-1}$, a rational expectations equilibrium is defined as a process $\left(C_t, L_t, X_t, Y_t, w_t, R_t, R_t^k, N_t, K_t, B_t, V_t, q_t, i_t, P_t, \tilde{p}_t\right)_{t \in \mathbb{N}}$ that solves the household problem, the retailer problem, satisfies the monetary policy rule, satisfies the following conditions in the intermediate firm sector:

$$
N_t = R_t^k K_{t-1} - R_t B_{t-1}
$$

$$
K_t = (1 - \gamma + \gamma \omega) N_t + B_t
$$

$$
B_t = \frac{\xi}{1 - \xi} V_t
$$

$$
V_t = \beta E_t \left[ \gamma N_{t+1} + \frac{(1 - \gamma)}{(1 - \gamma + \gamma \omega)} V_{t+1} \right]
$$

$$
R_t^k = \alpha q_t \left( \frac{(1 - \alpha)}{\bar{w}_t} q_t A_t \right) \frac{1 - \alpha}{\alpha} + 1 - \delta
$$

$$
w_t = q_t (1 - \alpha) \frac{Y_t}{N_t}
$$

$$
Y_t = K_t^\alpha (A_t L_t)^{1 - \alpha}
$$

and clears the markets for stocks and consumption goods:

$$
S^j_t = 1 \forall j
$$

$$
Y_t = \int_0^1 C_t^d i + C_t^c + K_t - (1 - \delta) K_{t-1}
$$

In this equilibrium, the borrowing constraint always binds and continuing firms never pay dividends. This is optimal in a neighbourhood of any non-stochastic balanced growth
path in which $R^k > R$.

I assume that productivity has a transient and a permanent component, the latter consisting in a random walk with non-stochastic drift, and that the monetary policy shock is an AR(1)-process:

$$
\begin{align*}
\log A_t &= \log A_{1t} + \log A_{2t} \\
\log A_{1t} &= \log A_{1t-1} + \log G + \varepsilon_{1t} \\
\log A_{2t} &= \rho \log A_{2t-1} \varepsilon_{2t} \\
\log \nu^m_t &= \rho_m \log \nu^m_{t-1} + \log \varepsilon^m_t
\end{align*}
$$

After stationarising the model to account for productivity growth, I log-linearise the equilibrium conditions around a balanced growth path. In particular, this means that a standard New-Keynesian Phillips curve emerges which takes the simple form

$$
\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \frac{1 - \alpha_p}{\alpha_p} (1 - \alpha_p \beta) \hat{q}_t
$$

Finally, the solution of the model under learning is a two-step procedure. I first need to compute the agents’ optimal plans given their beliefs under anticipated utility. To this end, I replace the asset pricing equation (9) with the belief system (12)-(14), which adds the state variable $\mu_t$ and adds two disturbances $\eta_t$ and $\nu_t$. I calculate the log-linearised solution to this system. The resulting policy rules can be used to calculate the actual stock market price $P_t$ from the Euler equation under learning (15), given a belief $\hat{\mu}_{t-1}$ about stock price growth. The difference between the expected price and the realised price is seen by the agents as a combination of a disturbance $\eta_t$ and a shift in the likely value of $\mu_t$, which can be computed from the belief updating equation.

### 4.2 Frictionless benchmark

Because my aim is to gauge the importance of financial frictions for business cycle analysis, I need a frictionless benchmark to which the model can be compared to. The benchmark model is identical to the one outlined above, except that intermediate firms are now owned by households, and face no frictions for access to external finance. The model then becomes identical to a standard New-Keynesian model with nominal rigidity, with the exception that the dividends and market value of intermediate producers do not include rents from monopoly power, as these are appropriated by retail firms.
5 Simulation results

5.1 Calibration

I calibrate the model parameters to match certain moments in postwar quarterly data on the economy of the United States. The non-stochastic trend productivity growth rate $G$ is set to its post-war average of 1.5% annually. The persistence of the temporary component of productivity is set to 0.99, the value most commonly used in business cycle studies. The capital share in production is set to $\alpha = 0.33$, implying a labour share in output of two thirds.

Turning to preferences, the intertemporal elasticity of substitution $\theta$ is set to unity, in line with most of the literature. The values for $G$ and $\theta$ imply a value for the household factor of $\beta = 0.9996$ in order to match the risk-free real rate of return in the US, which has been about 2% on average for 1-year treasuries. So $R^4 = 1.02$, which is the rate at which firms can borrow. I set the corresponding discount factor for entrepreneurs to match this rate of return $\tilde{\beta} = R^{-1/4}$. The parameter $\phi$ is the inverse Frisch elasticity of labour supply. There is no real agreement of what this elasticity should be. Peterman (2012) reports that microeconomic estimates range from 0 to 0.5, while macroeconomic models typically calibrate to values between 2 and 4. He suggests a value of 3 for macroeconomic applications, which corresponds to setting $\phi = 0.33$ in my model.

Next, the parameters describing the financial constraints of intermediate producers are $\gamma$, the rate of firm exit and entry; $\xi$, the tightness of the borrowing constraint; and $\omega$, the equity received by young firms relative to average equity. I set these three parameters to match the US investment share in output of 18% and the quarterly price-dividend ratio of 140; while maintaining stationarity. Intuitively, a higher rate of exit $\gamma$ makes firms shorter-lived, reducing the firm’s value relative to its current dividend payments. So a higher value of $\gamma$ induces a lower P/D ratio. A higher value of $\xi$ relaxes the borrowing constraint, which increases investment and, since returns on capital are decreasing, raises the investment share in output. Finally, there is only one value of $\omega$ for which a stationary equilibrium exists. If young firms are endowed with too much equity, they are able to acquire external funds so fast that they eventually become unconstrained; if they are endowed with too little, they cannot cover their debt costs and their net worth shrinks to zero. The parameter values chosen are $\gamma = 0.0501$, $\xi = 0.1333$ and $\omega = 0.1524$.

Turning to the nominal rigidity side of the model, I set the degree of price stickiness $\alpha = 0.75$, corresponding to an average time between price adjustments of 3 quarters. The elasticity of substitution between varieties of the final consumption good $\sigma$ is set to achieve a markup over marginal cost of 30%. The aggressiveness of monetary policy with respect to inflation is set to $\phi_r = 1.5$ and the persistence of monetary shocks is set to $\rho_m = 0.5$.

The only parameter left is the learning gain $g$. In principle, it could be calibrated to match data on expectations about stock price growth. Since the aim of the paper is to assess
the impact of endogenous asset price volatility on credit frictions, I choose to calibrate the learning gain such that the volatility of the stock market matches that of the data. More precisely, I match the volatility of stock market prices relative to the volatility of dividends to be 2.59, which is the corresponding post-war value of the S&P1500.

5.2 Discussion

Figure 5 plots the impulse responses to a unit-size persistent technology shock under rational expectations, under learning and in the frictionless comparison. It is immediate that the model under rational expectations does not behave very differently from the frictionless case. The effect of technology shocks are slightly amplified, but not markedly so. By contrast, the learning solution generates a large amount of amplification and propagation. The effects of a positive productivity shock build up quite slowly, both due to slow accumulation of net worth and slow adjustment of beliefs about permanent stock market growth. They then reverse and even undershoot slightly. Output rises by four times as much as TFP in the process. Moreover, it produces a hump-shaped response of investment and consumption without relying on adjustment costs.

The amplification of the model is due to a strong positive feedback loop between beliefs in the financial market and fundamentals in the economy. The nature of this feedback loop can be best understood in a diagram such as Figure 7. Consider any positive shock to net worth $N_t$. This increases the internal resources of the firm available for investment, so $K_t$
Figure 6: Impulse responses to a monetary shock.

t = 0

$N_t$ rises

$D_t$ rises

$t = 1$

$K_t$ rises

$D_{t+1}$ rises

$P_t$ rises

$t = 2$

$N_{t+1}$ rises

$K_{t+1}$ rises

$N_{t+2}$ rises

$P_{t+1}$ rises

$D_{t+2}$ rises

$\hat{\mu}_t$ rises

$\hat{\mu}_{t+1}$ rises

Figure 7: Feedback between beliefs and fundamentals.
rises. If the general equilibrium effects of rising wages and interest rates do not outweigh the increased profits from reduced financial frictions, then this also raises next worth in the next period $N_{t+1}$. This increases firm value today and permits more borrowing, as firms can pledge increased profits from the next period as collateral for their loans. By consequence, investment rises even more. This forward-looking channel is the standard financial accelerator effect, even though it relates to the value of equity instead of the value of capital or land.

However, there is a second amplification channel interacting with the first one that stems from changes in beliefs. Under rational expectations, equity value is not volatile, and therefore its effect on investment is not large. Under learning, however, agents are unsure to what extent the increase in $P_t$ is temporary (is due to $\eta_t$) or signals an increase in the long-run growth of the market (is due to $\nu_t$). They update their belief $\hat{\mu}_{t+1}$ upwards in the next period. In order for the equity market to clear, the price $P_{t+1}$ rises in next period as well. Hence investors’ expectations $\hat{\mu}_{t+2}$ rise even further etc. Gradually, expectations and prices rise. This is the momentum effect as outlined in Adam and Marcet (2011). Each time the stock market valuation increases, firms experience easier borrowing, can invest more and become more profitable, endogenously paying out higher dividends. This reinforces the upswing in the stock market, and it is in this sense that beliefs and fundamentals form a positive feedback loop.

To get an idea about how much of the stock market increase is due to learning itself, and how much due to the feedback arising from relaxed borrowing constraints and higher dividends, Figure 8 plots the impulse responses of the stock market value $P_t$ to a productivity shock in the rational expectations model, the learning model and in a hypothetical world where learning takes place, but dividends are kept at their path under rational expectations. This counterfactual price displays the same qualitative behaviour, but the magnitude of the response to the shock is much lower. Based on this comparison, the positive feedback loop accounts for about 40% of stock market fluctuation in the model.
6 Conclusion

The present paper has shown the importance of asset price fluctuations in a model of credit frictions. In a model where firms borrow against the market value of their assets, learning in the stock market prices combines with credit constraints to generate a powerful financial accelerator that substantially amplifies supply and demand shocks for reasonably calibrated model parameters. The key mechanism at play is a feedback loop between beliefs, prices and fundamentals, by which high stock prices relaxing firms’ borrowing constraints, allowing them to increase their investment and profits, which in turn induces optimistic belief and raises stock prices even more. Under rational expectations, the low volatility of stock prices prevents amplification.

Thus, the mechanism outlined here is one possibility to reduce the reliance on large exogenous shocks to explain cyclical fluctuations. However, further steps need to be taken to examine its empirical validity. First, the link between the stock market and credit constraints has to be examined. One prediction of the model is that the preference of lenders to restructure firms rather than to liquidate them in the event of default should depend on the aggregate stock market valuation. This is a testable hypothesis that can be examined empirically. Second, the present model can be readily estimated using standard Bayesian techniques for log-linearised models. It can then be established quantitatively to what extent learning reduces the variance of underlying shocks required to match the data.

On the theoretical side, more discipline has to be put on the calibration, as some aspects of the model behaviour are unsatisfactory. The volatility of consumption and hours worked relative to output is rather high, while that of investment is too low. Moreover, dividends and stock prices are still an order of magnitude less volatile than in the data. Finally, the learning scheme might be seen as overly simplistic. Survey data on expectations both of financial and real variables could be used to determine the expectations formation process.
References


A Appendix

A.1 Linearity of firm value in net worth

Let $V$ be the value of a firm in a given period before entry and exit. At this time, the firm finds itself with capital $K$ and debt $B$ (subscripts $j$ indexing firms are omitted here). Its value depends on these state variables and the aggregate state of the economy, denoted by $s$. After choosing its labour demand and realising production, the firm finds itself with net worth $N$. It then exits the economy with probability $\gamma$, paying out all its net worth. New firms arrive with new initial net worth. Let $W$ be the value of a firm after entry and exit; this only depends on its net worth and the aggregate state. We have

$$V(K, B, s) = \max_{N, L} \gamma N + (1 - \gamma) W(N, s)$$

s.t. $N = qK^\alpha (AL)^{1-\alpha} - wL + (1 - \delta) K - RB$

and

$$W(N, s) = \max_{D, K, B} D + \beta E [V(K, B, s') | s]$$

s.t. $K + D = N + B$

$B \leq \max (\xi K, \beta E [V(\xi K, \xi B, s') | s] + \xi B)$

$D \geq 0$
The problem for \( V \) can be solved easily. The first-order condition with respect to \( L \) is 
\[
(1 - \alpha) q \left( \frac{K}{L} \right)^\alpha A^{1-\alpha} = w,
\]
and so every firm chooses the same capital-labour ratio. Consequently, \( R_k \) as defined in the text is well-defined and
\[
N = R_k K - RB
\]
\[
V(K, B, s) = V(N, s) = \gamma N + (1 - \gamma) W(N, s)
\]

Next, write the Lagrangean for the value function \( W \). With a slight abuse of notation, I denote next period’s net worth by \( N' = R_k - RB \):
\[
W = \max_{D, K, B, \lambda, \mu, \eta} (1 + \eta) D + \lambda (N + B - K - D) + \mu \left( \max (\xi K, \beta E[V(\xi N', s') | s] + \xi B) - B \right) + \beta E[V(\xi N', s') | s]
\]
The first-order conditions are
\[
-\lambda + \mu \xi I + \beta \xi (1 - I) E \left[ \frac{\partial V}{\partial N}(\xi N', s') R_k | s \right] + \beta E \left[ \frac{\partial V}{\partial N}(N', s') R_k | s \right] = 0
\]
\[
\lambda - \mu - \mu \xi (1 - I) \left( \beta E \left[ \frac{\partial V}{\partial N}(\xi N', s') R | s \right] - 1 \right) - \beta E \left[ \frac{\partial V}{\partial N}(N', s') R | s \right] = 0
\]
where the variable \( I \) takes the value one whenever liquidation is preferred to restructuring \( (\xi K > \beta E[V(\xi N', s')]) \) and zero otherwise. These conditions are supplemented by complementary slackness conditions.

I conjecture that firm value takes a linear form in net worth, i.e. that \( V(N, s) = UN \) where \( U \) only depends on the aggregate state and where \( U \geq 1 \). The linearity in net worth ensures that firms can be aggregated across, as in Bernanke, Gertler, and Gilchrist (1999). The guess is verified by backward induction, i.e. by solving for \( W(N, s) \) assuming the guess for \( V(N', s') \), and verifying that \( \partial W/\partial N \) is independent of \( N \) and greater or equal than unity. The same properties then hold for \( V \). This will establish that the conjectured form for firm value is consistent with firm optimisation. That it is also part of an equilibrium of the model will be shown only for the non-stochastic steady-state, directly by constructing it.

So assume \( V(N', s') = U' N' \). The first-order conditions for \( W \) can then be rewritten as
\[
1 + \eta = \lambda
\]
\[
\mu + \beta (\xi \mu (1 - I) + 1) E[U'R | s] = \lambda
\]
\[
\mu \xi + \beta (\xi \mu (1 - I) + 1) E\left[U' \left( R_k - R \right) | s \right] = \mu
\]
From condition (19), it is immediate that \( \lambda \geq 1 \) since \( \eta \geq 0 \). Further, by the envelope
theorem, we have
\[
\frac{\partial W}{\partial N} = \lambda
\]
What needs to be shown next is that \( \lambda \) does not depend on the value of \( N \). To do this, I first look at the value of \( \mu \). The first order condition (21) can be rewritten as
\[
\beta E \left[ U' \left( R^k - R \right) \mid s \right] = \mu \left( 1 - \xi \right) \frac{1 + \mu \xi I}{1 + \mu \xi}
\]
Note that the multiplier \( \mu \) has to be non-negative. From this, it is immediate that \( E \left[ U' \left( R^k - R \right) \mid s \right] \geq 0 \) in equilibrium. That is, the internal return on investment has to be at least as high as the one on debt. Further, no matter what the value of \( I \), we have \( \mu > 0 \) if and only if \( E \left[ U' \left( R^k - R \right) \mid s \right] > 0 \), which only depends on the aggregate state. So either all firms are borrowing-constrained or all firms are unconstrained. In the case where they are all unconstrained, we have \( \lambda = E \left[ U' R \mid s \right] \) which only depends on the aggregate state.

For the other case, in which all firms are constrained, I require an additional assumption, which is that \( \beta E \left[ U' R \right] \) be larger than unity. As mentioned in the text, I consider parameter values for which \( \beta R = 1 \) holds in the non-stochastic steady state, so since \( U \geq 1 \) this condition is satisfied locally around the steady-state. Using this, we can make \( \lambda \) large in the first-order condition (20):
\[
\lambda = \mu + \beta \left( \xi \mu \left( 1 - I \right) + 1 \right) E \left[ U' R \mid s \right] > 1
\]
and thus \( \eta > 0 \), i.e. no firm pays dividends as long as it is financially constrained.

Now, the value of \( I \) is still undetermined and could in principle vary across firms, i.e. lenders to some firms might prefer restructuring over liquidation and others might not. However, this is not the case. Suppose that for one particular firm (one particular value of \( N \)), \( I = 1 \). This entails \( B = \xi K \). The fact that \( I = 1 \) then means that
\[
\xi K < \xi \beta E \left[ U' N \right] + \xi B = \xi \beta E \left[ U' \left( R^k K - R K \right) \right] + \xi^2 K
\]
\[
\Leftrightarrow \xi \left( \beta E \left[ U' R \right] - 1 \right) < \beta E \left[ U' R \right] - 1
\]
On the other hand, if for some value of \( N \), we have \( I = 0 \), then this entails

\[
\begin{align*}
B &= \xi B + \xi \beta E [U'N'] \\
&= \xi B + \xi \beta E [U'R^k] K - \xi \beta E [U'R] B \\
&= \frac{\xi \beta E [U'R^k]}{1 - \xi + \xi \beta E [U'R]} K \equiv XK
\end{align*}
\]

The fact that \( I = 0 \) then means that

\[
\xi K \geq \xi \beta E [U'N'] + \xi B \\
\Leftrightarrow X (\beta E [U'R] - 1) > \beta E [U'R^k] - 1
\]

Now, given the assumption \( \beta E [U'R] \geq 1 \) used previously, and as we also must have \( E [U'R^k] > E [U'R] \) since \( \mu > 0 \), we have \( X > \xi \). Therefore, it is not possible that the conditions for \( I = 0 \) and \( I = 1 \) hold at the same time, and so all firms need to have the same value of \( I \). This in turn implies that the values of \( \mu \) and \( \lambda \) must also be equal across firms by conditions (20) and (21). Therefore, we have \( \partial W/\partial N = \lambda \geq 1 \) and independent of \( N \). Finally,

\[
\frac{\partial V}{\partial N} = \gamma + (1 - \gamma) \frac{\partial W}{\partial N} \\
= \lambda + (1 - \gamma) \lambda \\
\geq 1
\]

which completes the proof that firm value that is linear in net worth and at least as large as net worth is consistent with firm optimisation.

### A.2 Preference of restructuring over liquidation

The outside option of a lender in a debt renegotiation process at time \( t \) is to either restructure or liquidate the firm. If she liquidates, then she gets a payoff of \( \xi K^j_t \) from the sale of the capital that remains of the defaulted firm. If she restructures, then she retains a fraction of her debt holdings and sells the firm with assets \( \xi K^j_t \) and debt \( \xi B^j_t \) to another entrepreneur. Such a firm, in period \( t + 1 \), will have net worth \( \tilde{N}^j_{t+1} = R^k_{t+1} \xi K^j_t - R_t \xi B^j_t = \xi N^j_{t+1} \) and, accordingly, be priced at \( \tilde{P}^j_{t+1} = \xi P^j_{t+1} \). The firm has already paid out dividends in the current period, so risk-neutral entrepreneurs are willing to buy the ex-dividend firm at time \( t \) at the price \( \beta E_t \tilde{P}^j_{t+1} \). Thus, the lender prefers restructuring over liquidation iff

\[
\xi K^j_t < \xi \beta E_t V^j_{t+1} + \xi \beta E_t \frac{\lambda_{t+1}}{\lambda_t} R_t B^j_t
\]
In the steady state under consideration here, all firms choose the same capital-debt ratio and the stock price is linear in net worth. Therefore, the above equation reduces to

\[ \tilde{K} < \beta E_t \tilde{V} + \tilde{B} \Rightarrow \beta \frac{\tilde{P}}{\tilde{K}} > 1 - \frac{\tilde{B}}{\tilde{K}} \]

In what follows, it will be convenient to express all quantities as a fraction of the capital stock \( \tilde{K} \). Starting with equation (9) for the equity value, it can be written in steady state as

\[ \frac{\tilde{V}}{\tilde{K}} = \frac{1}{1 - \beta} \left( 1 - \frac{(1 - \gamma) \tilde{N}}{(1 - \gamma) \tilde{N} + \gamma \omega} \right)^{-1} \beta \tilde{N}/\tilde{K}, \]

and net worth before and after exit are

\[ \frac{\tilde{N}}{\tilde{K}} = \tilde{R}^k - \tilde{R}^B_K \]

and

\[ \left( (1 - \gamma) \tilde{N} + \gamma \omega \right) / \tilde{K} = 1 - \tilde{B} / \tilde{K}. \]

Substituting into the equity value equation and noting that \( \beta = R^{-1} \):

\[ \beta \frac{\tilde{V}}{\tilde{K}} = \left( 1 - \beta \frac{(1 - \gamma) (\tilde{R}^k - \tilde{R}^B_K)}{1 - \frac{\tilde{B}}{\tilde{K}}} \right)^{-1} \gamma \beta \left( \tilde{R}^k - \tilde{R}^B_K \right) \]

\[ = \left( 1 - \frac{\tilde{B}}{\tilde{K}} \right) \beta \gamma \left( \tilde{R}^k - \tilde{R}^B_K \right) \frac{1 - \frac{\tilde{B}}{\tilde{K}} - \beta (1 - \gamma) \left( \tilde{R}^k - \tilde{R}^B_K \right)}{1 - \frac{\tilde{B}}{\tilde{K}}} \]

\[ = \left( 1 - \frac{\tilde{B}}{\tilde{K}} \right) \gamma \left( \tilde{R}^k - \tilde{R}^B_K \right) \frac{R - \tilde{R}^B_K - (1 - \gamma) \left( \tilde{R}^k - \tilde{R}^B_K \right)}{R - \tilde{R}^B_K - (1 - \gamma) \left( \tilde{R}^k - \tilde{R}^B_K \right)} \]

\[ > \left( 1 - \frac{\tilde{B}}{\tilde{K}} \right) \gamma \left( \tilde{R}^k - \tilde{R}^B_K \right) \frac{\tilde{R}^k - \tilde{R}^B_K - (1 - \gamma) \left( \tilde{R}^k - \tilde{R}^B_K \right)}{\tilde{R}^k - \tilde{R}^B_K - (1 - \gamma) \left( \tilde{R}^k - \tilde{R}^B_K \right)} \]

\[ = 1 - \frac{\tilde{B}}{\tilde{K}} \]

The inequality is thus established using only the condition \( \tilde{R}^k > \tilde{R} \), which needs to hold whenever firms are financially constrained.